



Gleniffer High School

Higher

Our Dynamic Universe

Summary Notes

Name: _____

Our Dynamic Universe

MOTION – Equations and Graphs

Vectors and Scalars

Physical quantities can be divided into two groups:

- A **scalar** quantity is completely described by stating its **magnitude** (size).
- A **vector** quantity is completely described by stating its **magnitude and direction**.

Distance is a **scalar** quantity. Distance is the total path length. It is fully described by magnitude (size) alone,

Displacement is a **vector** quantity. Displacement is the direct length from the starting point to the finishing point. To fully describe displacement both magnitude and direction must be given.

Speed is another example of a **scalar** quantity. **Velocity** is another example of a **vector** quantity. Speed and velocity are described by the equations below.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

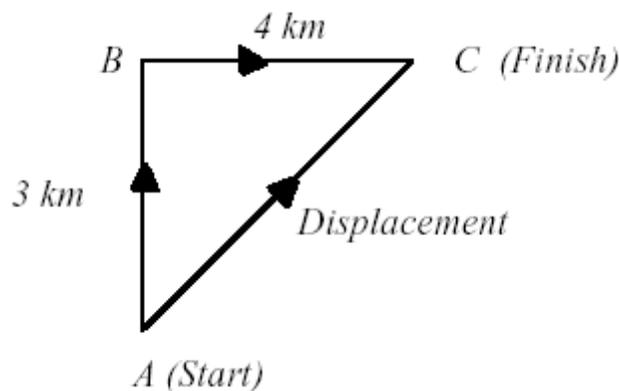
$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

The direction of the velocity will be the same as the direction of the displacement.

Example

A woman walks 3km due North, and then 4km due East. This takes her 2hours. Find her:

- a) distance travelled.
- b) displacement from her starting point.
- c) average speed.
- d) average velocity.



Solution

a)

Distance travelled = (3 + 4) km = 7km

b)

Displacement could be found by using a scale diagram or Pythagoras to get the magnitude and SOHCAHTOA to get the bearing.

Displacement = 5km on a bearing of 053.

c)

average speed = distance/time

average speed = $7/2$

average speed = 3.5kmh^{-1}

d)

average velocity = displacement/time

average velocity = $5/2$

average velocity = 2.5kmh^{-1} on a bearing of 053

A summary table of common scalars and vectors is shown below.

Vectors	Scalars
displacement	distance
velocity	speed
acceleration	time
force	mass
impulse	energy

The Equations of Motion

The equations of motion for objects with constant acceleration in a straight line are shown below.

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

where,

u – is the initial velocity of the object, measured in ms^{-1}

v – is the final velocity of the object, measured in ms^{-1}

a – is the acceleration of the object, measured in ms^{-2}

s – is the displacement of the object, measured in m

t – is the time for which the motion takes place, measured in s

It should also be noted that **u, v, a and s are all vector quantities** and that a positive direction is chosen when carrying out calculations.

Example 1

A skier, starts from rest, and accelerates at 2.5ms^{-2} down a straight ski run. Calculate her velocity after 9seconds.

$$\begin{aligned}u &= 0\text{ms}^{-1} \\v &= ? \\a &= 2.5\text{ms}^{-2} \\s &= ? \\t &= 9\text{s}\end{aligned}$$

$$\begin{aligned}v &= u + at \\v &= 0 + 2.5 \times 9 \\v &= 22.5\text{ms}^{-1}\end{aligned}$$

Example 2

A train is travelling at 3ms^{-1} when it begins to accelerate at 2ms^{-2} . The displacement of the train during the acceleration is 54m. Calculate the final velocity of the train.

$$\begin{aligned}u &= 3\text{ms}^{-1} \\v &= ? \\a &= 2\text{ms}^{-2} \\s &= 54\text{m} \\t &= ?\end{aligned}$$

$$\begin{aligned}v^2 &= u^2 + 2as \\v^2 &= (3)^2 + 2 \times 2 \times 54 \\v^2 &= 9 + 216 \\v^2 &= 225 \\v &= 5\text{ms}^{-1}\end{aligned}$$

Example 3

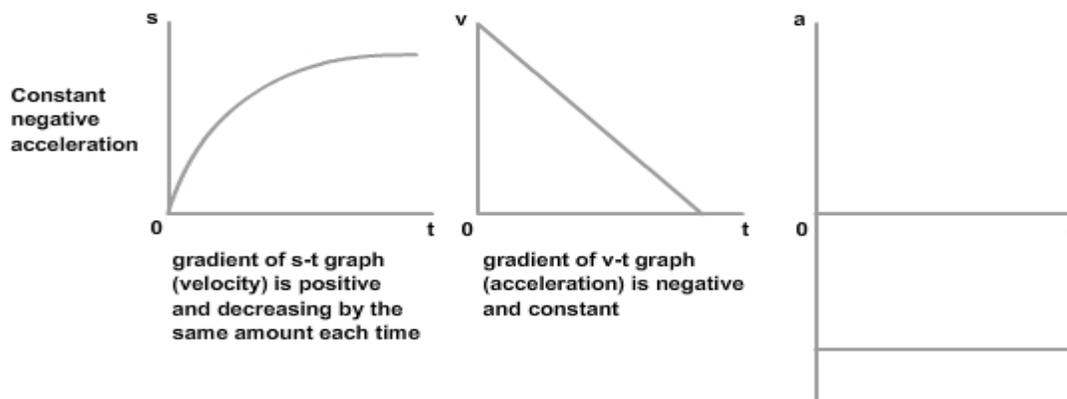
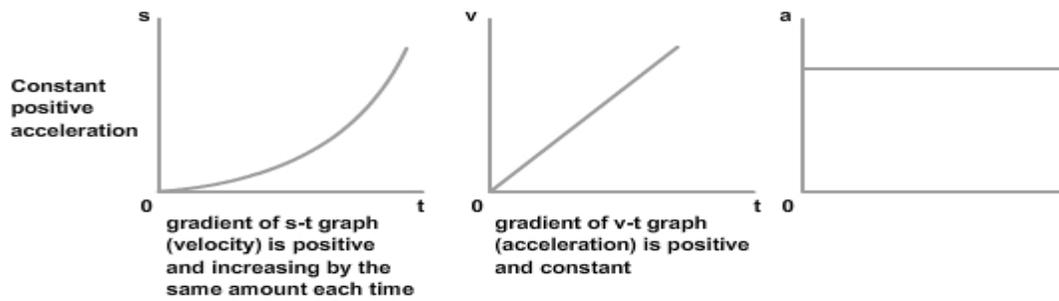
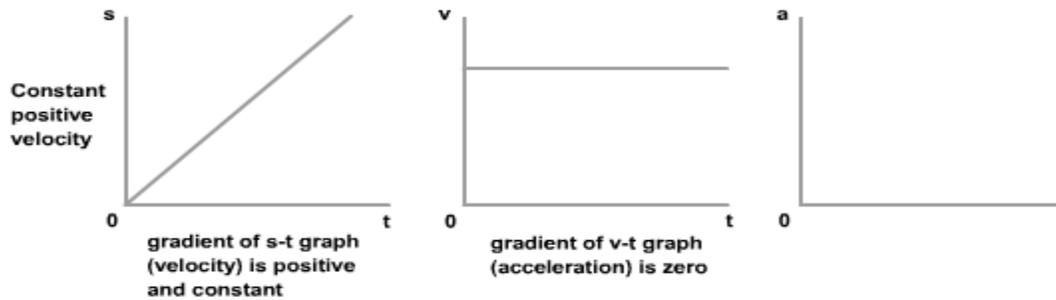
An object accelerates at a constant rate of 2ms^{-2} , from an initial velocity of 4ms^{-1} . How far does the object travel in 5seconds?

$$\begin{aligned}u &= 4\text{ms}^{-1} \\v &= ? \\a &= 2\text{ms}^{-2} \\s &= ? \\t &= 5\text{s}\end{aligned}$$

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\s &= (4 \times 5) + (0.5 \times 2 \times (5)^2) \\s &= 20 + 25 \\s &= 45\text{m}\end{aligned}$$

Motion Graphs

Motion-time graphs for motion with constant acceleration are shown below.



Notes

The gradient of a displacement-time graph gives the velocity of the motion.

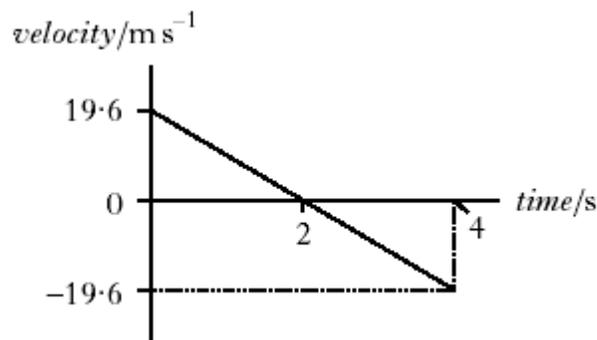
The area under a velocity-time graph gives the displacement.

The gradient of a velocity-time graph gives the acceleration.

Special Graphs

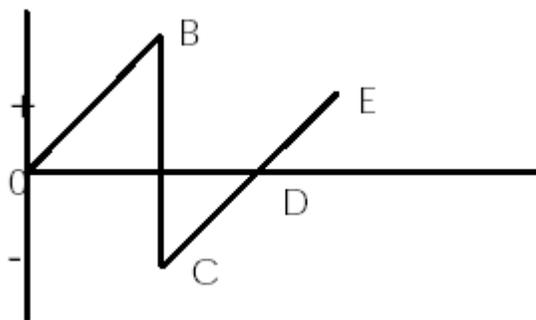
The motion of an object thrown upwards could be represented by the following v vs t graph. (The graph assumes upwards motion is positive.)

The object has been thrown with an upwards velocity of 19.6ms^{-1} and reaches its highest point after 2seconds. (Vertical velocity is zero.) After a further 2seconds the object returns to its starting point but is travelling with a downwards velocity of 19.6ms^{-1} .



The motion of a ball which is dropped could be represented by the following v vs t graph. (The graph assumes downwards motion is positive.)

The ball accelerates downwards between O and B, where it strikes the ground. The ball bounces and leaves the ground at C. Between C and D the ball is travelling upwards and reaches its highest point at D. (Vertical velocity is zero.) Between D and E the ball is returning to the ground.



Falling Objects

All falling objects are subjected to the gravitational attraction of the Earth and will accelerate downwards. In motion calculations it can be assumed that the value of this acceleration is 9.8ms^{-2} . It can also be assumed that the vertical velocity of a falling object will be zero at the highest point of its motion.

Example

A ball is dropped from a height of 66metres.

- How long does it take to hit the ground?
- What is the velocity of the ball just before it strikes the ground?

a)

$$u = 0\text{ms}^{-1}$$

$$v = ?$$

$$a = 9.8\text{ms}^{-2}$$

$$s = 66\text{m}$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$66 = (0 \times t) + (0.5 \times 9.8 \times (t)^2)$$

$$66 = 0 + 4.9t^2$$

$$66 = 4.9t^2$$

$$t^2 = 66/4.9$$

$$t = 3.7\text{s}$$

b)

$$u = 0\text{ms}^{-1}$$

$$v = ?$$

$$a = 9.8\text{ms}^{-2}$$

$$s = 66\text{m}$$

$$t = 3.7\text{s}$$

$$v = u + at$$

$$v = 0 + 9.8 \times 3.7$$

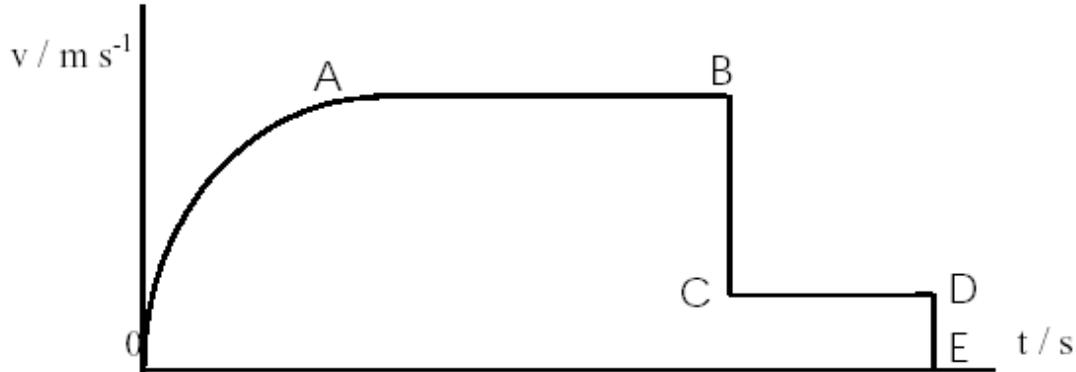
$$v = 36.3\text{ms}^{-1}$$

The table below contains the details of the Past Paper examples for this area of the course. Past Papers, and their solutions, are free to download from the SQA website.

Year	Section One	Section Two
2015	1 and 2	2b)iii
2016	1 and 2	No examples
2017	1	1a) and 3
2018	1 and 2	No examples
2019	1 and 4	1 a) and c)

FORCES, ENERGY and POWER

Shown below is the graph of a parachutist from the moment they leave the plane until they reach the ground.



The shapes on the graph can be explained using **Newton's Laws of Motion**.

From O to A

There is an unbalanced force acting on the parachutist causing an acceleration. This is in agreement with Newton's 2nd Law.

From A to B

There are balanced forces present and there is a constant velocity. This is in agreement with Newton's 1st Law.

From C to D

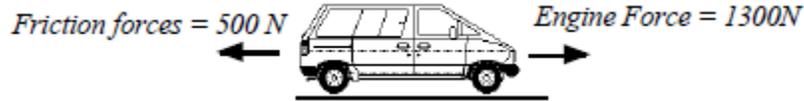
Assuming the parachute is deployed at point B, there is a lower constant velocity for this part of the graph due to the increased air resistance. Note that Newton's 1st Law still applies and this velocity is called the terminal velocity.

Force Calculations

For all force calculations it is useful to draw a diagram of all the forces acting on the object. The forces should be represented by arrows. This is known as a free body diagram.

Example 1

A car of mass 1000kg experiences a frictional force equal to 500N. If the engine force is 1300N, what will be the car's acceleration?

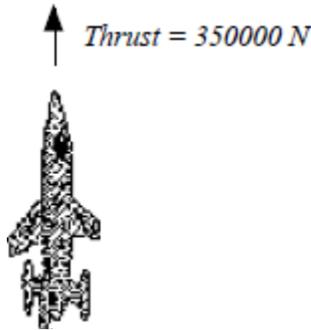


$$F_{\text{unb}} = 800\text{N} (1300 - 500)$$
$$m = 1000\text{kg}$$
$$a = ?$$

$$F = ma$$
$$800 = 1000 \times a$$
$$a = 800/1000$$
$$a = 0.8\text{ms}^{-2}$$

Example 2

The rocket shown below has a mass of 10,000kg at lift-off. Calculate the rocket's initial acceleration.



To calculate the F_{unb} we need to know the downward force.

Step One

$$\text{Downward force} = \text{Weight} = ?$$
$$m = 10,000\text{kg}$$
$$g = 9.8\text{Nkg}^{-1}$$

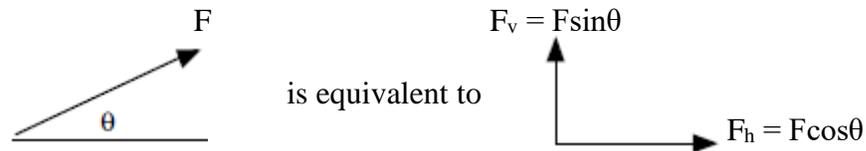
$$W = m \times g$$
$$W = 10,000 \times 9.8$$
$$W = 98,000\text{N}$$

Step Two

$$F_{\text{unb}} = F_{\text{up}} - F_{\text{down}}$$
$$F_{\text{unb}} = 350,000 - 98,000$$
$$F_{\text{unb}} = 252,000\text{N}$$

$$F_{\text{unb}} = m \times a$$
$$252,000 = 10,000 \times a$$
$$a = 252,000 / 10,000$$
$$a = 25.2\text{ms}^{-2}$$

Forces are often applied at an angle. But as force is a vector quantity it can be resolved into two **components at right angles** to each other. These components are sometimes called **rectangular components**.

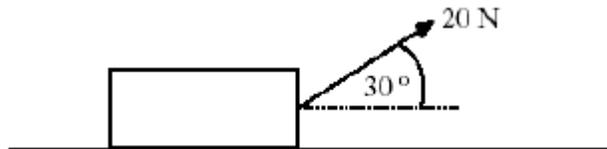


The above diagrams show that the force, F , applied at an angle Θ can be resolved into two components.

- A horizontal component $F_H = F \cos \Theta$.
- A vertical component $F_V = F \sin \Theta$.

Example

A force is applied to a box as shown below. The box moves with a constant speed along the surface.



Calculate the force of friction acting on the box when it is moving along the surface.

Solution

As the box moves with a constant speed Newton's 1st Law can be applied. This means the force of friction must be equal to the horizontal component of the applied force.

$$F_H = F \cos \Theta.$$

$$F_H = 20 \cos 30$$

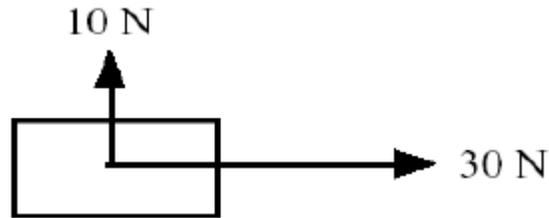
$$F_H = 17.3 \text{ N.}$$

The force of friction is 17.3 N.

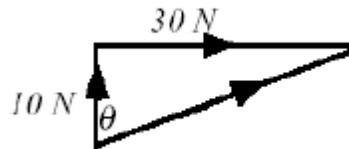
More than one force can be applied to an object at the same time. To add forces together in these situations magnitude and direction must be taken into account. This can be done by using a **“tip” to “tail” addition** followed by the use of appropriate **trig calculations**. (Scale diagrams can also be used to solve this type of force problem.)

Example

What is the resultant force acting on the object shown below?



The “tip to “tail” solution for this problem gives the following:



Magnitude by Pythagoras

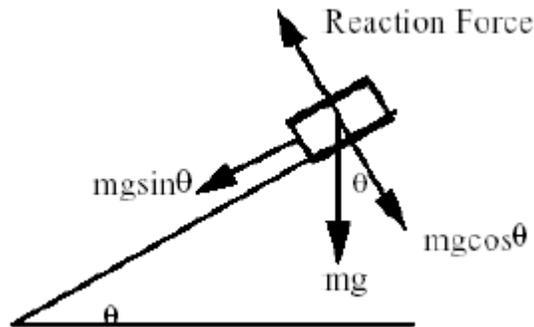
Direction by SOHCAHTOA

$$\begin{aligned} a^2 &= b^2 + c^2 \\ a^2 &= (10)^2 + (30)^2 \\ a^2 &= 100 + 900 \\ a^2 &= 1000 \\ a &= 31.6 \end{aligned}$$

$$\begin{aligned} \tan \Theta &= \text{opp} / \text{adj} \\ \tan \Theta &= 30 / 10 \\ \tan \Theta &= 3 \\ \Theta &= 72^\circ \end{aligned}$$

Resultant force = 31.6N on a bearing of 072

If an object is placed on a slope then its weight acts vertically downwards. As weight is a force, and therefore a vector quantity, it can be resolved into two components at right angles to each other. One component will act down the slope and the other component will act onto (perpendicular to) the slope.

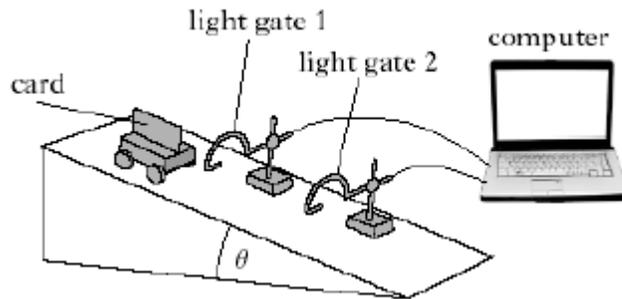


Component of weight down slope = $mg \sin \theta$

Component of weight perpendicular to slope = $mg \cos \theta$

Example

The apparatus shown below allowed the acceleration of the vehicle (mass 2kg) to be calculated as 2.9ms^{-2} . If the frictional force acting up the slope is 4N, calculate the angle of the slope.



$$F_{\text{unb}} = ma$$

$$F_{\text{unb}} = 2 \times 2.9$$

$$F_{\text{unb}} = 5.8\text{N}$$

$$F_{\text{unb}} = 5.8\text{N}$$

$$F_{\text{down}} - F_{\text{friction}} = 5.8$$

$$F_{\text{down}} - 4 = 5.8$$

$$F_{\text{down}} = 9.8\text{N}$$

$$F_{\text{down}} = 9.8\text{N}$$

$$mg \sin \Theta = 9.8$$

$$2 \times 9.8 \times \sin \Theta = 9.8$$

$$19.6 \times \sin \Theta = 9.8$$

$$\sin \Theta = 0.5$$

$$\Theta = 30^\circ$$

Conservation of Energy

The total energy of a closed system must be conserved, although the energy may change from one type into another type.

The equations for calculating kinetic energy (E_k), gravitational potential energy (E_p) and work done are given below.

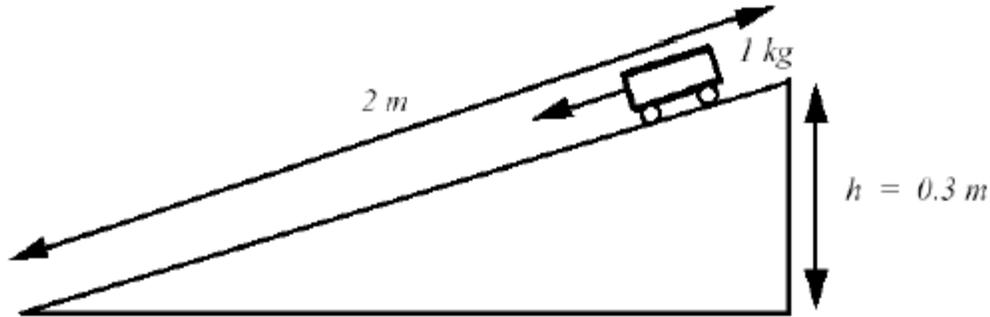
$E_p = mgh$	$E_k = \frac{1}{2}mv^2$	work done = force \times displacement
-------------	-------------------------	---

Energy and work done are measured in the same units, Joules (J).

Example

A trolley is released down a slope from a height of 0.3m. If its speed at the bottom is 2ms^{-1} , calculate:

- the work done against friction
- the force of friction on the trolley.



a)

At top,

$$E_p = mgh$$

$$E_p = 1 \times 9.8 \times 0.3$$

$$E_p = 2.94\text{J}$$

At bottom,

$$E_k = \frac{1}{2}mv^2$$

$$E_k = 0.5 \times 1 \times 2^2$$

$$E_k = 2.0\text{J}$$

As the energy at the top does not equal the energy at the bottom, by conservation of energy, some energy must have been used against friction. Therefore,

$$\text{work done} = (2.94 - 2.0) = 0.94\text{J}$$

b)

$$\text{work done} = \text{force} \times \text{displacement}$$

$$0.94 = \text{force} \times 2$$

$$\text{force} = 0.47\text{N}$$

When dealing with energies, power can be a useful quantity to remember.

Power is the rate of energy transformation. Power is measured in Watts (W).

$P = \frac{\text{energy}}{\text{time}} = \frac{\text{work done}}{\text{time}} = \frac{F \times \text{displacement}}{t} = F \times \text{average velocity}$
--

The table below contains the details of the Past Paper examples for this area of the course. Past Papers, and their solutions, are free to download from the SQA website.

Year	Section One	Section Two
2015	3, 4, 5 and 6	No examples
2016	3	2c)
2017	2 and 3	No examples
2018	3 and 4	2 not a)ii
2019	5	2

COLLISIONS, EXPLOSIONS and IMPULSE

Momentum and Impulse

Momentum is a useful quantity to consider when dealing with collisions and explosions. The momentum of an object is the product of its mass and velocity.

Momentum = mass \times velocity of the object.

A diagram showing the equation $\text{Momentum} = mv$ enclosed in a rectangular box. Three arrows point from the units below to the terms in the equation: an arrow from kg m s^{-1} points to m , an arrow from kg points to v , and an arrow from m s^{-1} points to mv .

Momentum is a vector quantity. The direction of an object's momentum is the same as the direction of the object's velocity.

Conservation of Linear Momentum

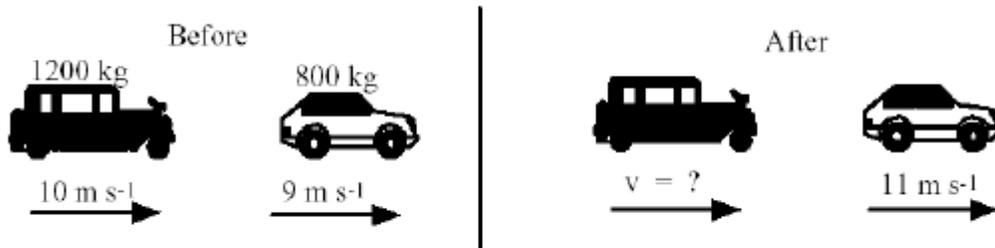
When two objects collide it can be shown that momentum is conserved provided there are no external forces applied to the system.

For any collision:

Total momentum of all objects before = total momentum of all objects after.

Example

One car approaches another from behind as shown below. Determine the speed of the rear car after the collision.



momentum before = momentum after

$$\begin{aligned}
 m_a v_a + m_b v_b &= m_{aa} v_{aa} + m_{ba} v_{ba} \\
 (1200 \times 10) + (800 \times 9) &= (1200 \times v_{aa}) + (800 \times 11) \\
 (12000) + (7200) &= (1200 v_{aa}) + (8800) \\
 19200 - 8800 &= 1200 v_{aa} \\
 10400 &= 1200 v_{aa} \\
 v_{aa} &= 8.7 \text{ms}^{-1}
 \end{aligned}$$

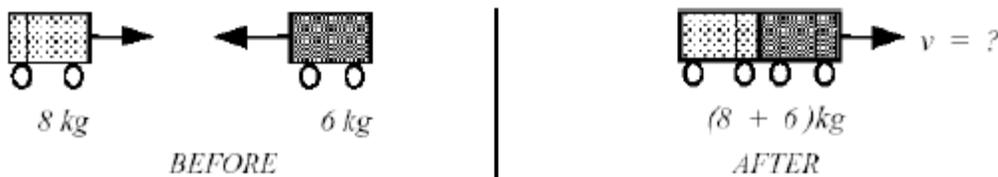
Collisions can be described as elastic or inelastic.

An **elastic** collision is one in which both **momentum and kinetic energy are conserved**.

An **inelastic** collision is one in which only **momentum is conserved**.

Example

In the collision shown below the 8kg trolley is moving to the right at 4ms⁻¹ before the collision. The 6kg trolley is moving to the left at 2ms⁻¹ before the collision. Calculate their combined velocity after the collision and show that this is an inelastic collision.



momentum before = momentum after

$$\begin{aligned}
 (m_a \times v_a) + (m_b \times v_b) &= (m_{\text{total}} \times v_{\text{total}}) \\
 (8 \times 4) + (6 \times -2) &= (14 \times v_{\text{total}}) \\
 32 + (-12) &= (14 \times v_{\text{total}}) \\
 20 &= (14 \times v_{\text{total}}) \\
 v_{\text{total}} &= 1.43 \text{ms}^{-1} \text{ to the right}
 \end{aligned}$$

E_k Before

$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$E_k = (0.5 \times 8 \times (4)^2) + (0.5 \times 6 \times (2)^2)$$

$$E_k = (64) + (12)$$

$$E_k = 76\text{J}$$

E_k After

$$E_k = \frac{1}{2}mv^2$$

$$E_k = 0.5 \times 14 \times (1.43)^2$$

$$E_k = 14.3\text{J}$$

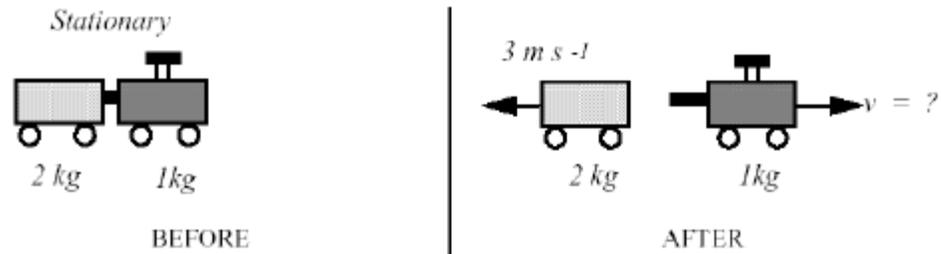
As the kinetic energy is not conserved this collision is inelastic.

Explosions

A single stationary object may explode into two parts. The conservation of linear momentum can be applied to this situation as well. The initial momentum will be zero (the object is stationary) so the final momentum will also be zero. Explosions are examples of **inelastic collisions** as the kinetic energy is not conserved.

Example

The trolleys shown below are exploded apart. Calculate the final velocity of the 1kg trolley.



Assume motion to the right is positive.

momentum before = momentum after

$$m_{\text{total}} \times v = m_a v_a + m_b v_b$$

$$3 \times 0 = (2 \times -3) + (1 \times v_b)$$

$$0 = -6 + v_b$$

$$v_b = 6 \text{ m s}^{-1} \text{ to the right}$$

Impulse

The impulse applied to an object is the average force exerted on the object multiplied by the time for which the force acts.

When an unbalanced force is exerted on an object it will accelerate and the following two equations can be applied to its motion.

$$F = ma \qquad v = u + at$$

The above equations can be combined as follows:

$$F = ma = \frac{m(v - u)}{t} = \frac{mv - mu}{t}$$

This leads to the following equation:

$$Ft = mv - mu$$

where,

Ft is the impulse measured in Ns

mv is the final momentum measured in kgms^{-1}

mu is the initial momentum measured in kgms^{-1}

This equation also shows that impulse is equal to the change in momentum.

Example

In a game of pool, the cue ball (mass 0.2kg), is accelerated, by a force from the cue, from rest to a velocity of 2ms^{-1} . The force is applied for a time of 50ms. What is the size of the force exerted by the cue?

$$Ft = mv - mu$$

$$F \times 50 \times 10^{-3} = (0.2 \times 2) + (0.2 \times 0)$$

$$F \times 50 \times 10^{-3} = 0.4$$

$$F = 8\text{N}$$

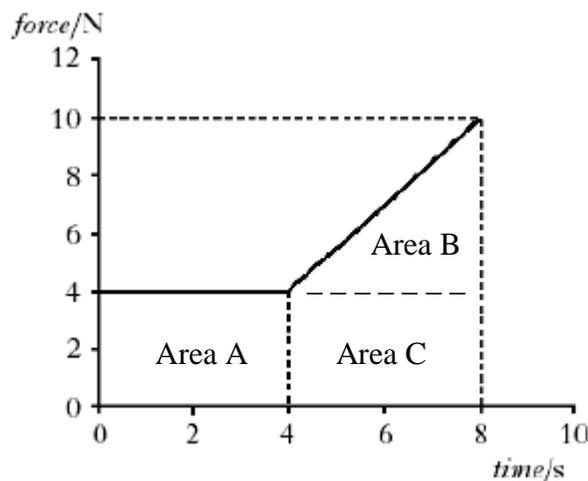
The concept of impulse is useful in situations where the force is not constant and acts for a very short period of time. An example is when a golf ball is struck by a golf club. During the contact the unbalanced force between the club and the ball varies as shown below.



Since F is not constant the impulse (Ft) is equal to the area under the graph. In any calculation involving impulse the unbalanced force that is calculated, is the average force. The maximum force experienced will be greater than the calculated average force.

Example

The graph below represents how the force on an object varies over a time interval of 8seconds.



Calculate the impulse applied to the object during the 8seconds.

$$\begin{aligned}\text{Impulse} &= \text{Area under a Ft graph} \\ \text{Impulse} &= (\text{Area A}) + (\text{Area B}) + (\text{Area C}) \\ \text{Impulse} &= (1 \times b) + (\frac{1}{2} \times b \times h) + (1 \times b) \\ \text{Impulse} &= (4 \times 4) + (0.5 \times 4 \times 6) + (4 \times 4) \\ \text{Impulse} &= (16) + (12) + (16) \\ \text{Impulse} &= 44\text{Ns}\end{aligned}$$

The table below contains the details of the Past Paper examples for this area of the course. Past Papers, and their solutions, are free to download from the SQA website.

Year	Section One	Section Two
2015	No examples	2 not b)iii
2016	4	3
2017	No examples	2
2018	No examples	3
2019	6	1 b)

GRAVITATION

Projectile Motion

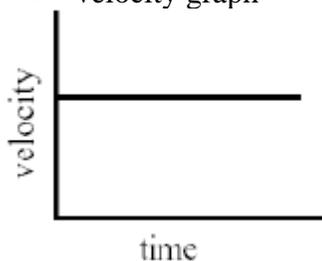
A projectile is an object which has been given a forward motion through the air, but which is also being pulled downward by the force of gravity. This results in the path of the projectile being curved.

A projectile has two separate motions at right angles to each other.

In calculations each motion must be treated independent of the other.

Horizontal

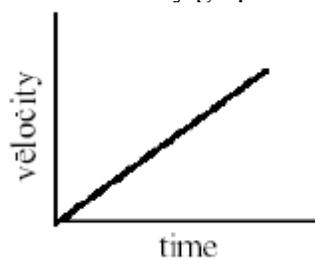
- constant speed
- for calculations use $d = v_h \times t$
- velocity graph



- component calculations
 $v_h = R \cos \Theta$

Vertical

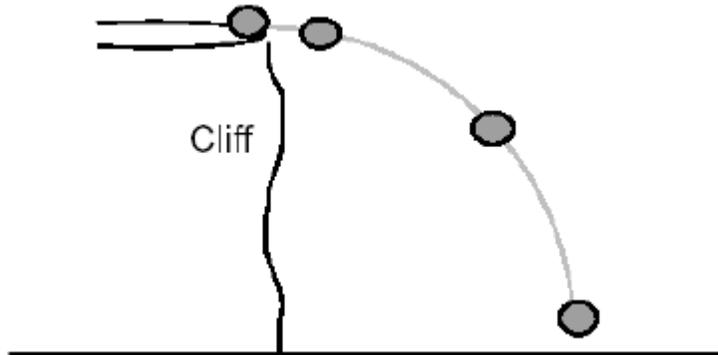
- constant acceleration
- for calculations use equations of motion
- velocity graph



- component calculations
 $v_v = R \sin \Theta$

Example One

A ball is kicked horizontally at 5ms^{-1} from the top of a cliff as shown below. It takes 2seconds to reach the ground.



- a) What horizontal distance did it travel in the 2seconds?
- b) What was its vertical velocity just before it hit the ground?

Solution

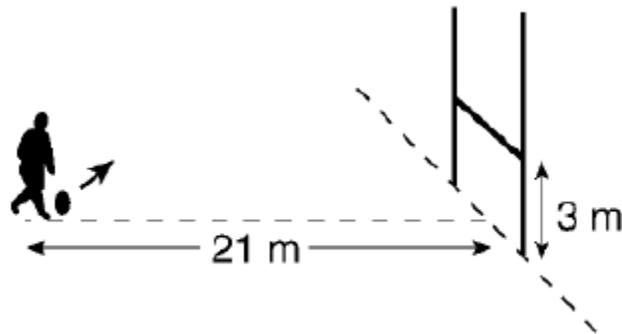
a)
 $v_h = 5\text{ms}^{-1}$
 $d = ?$
 $t = 2\text{s}$

$d = v_h \times t$
 $d = 5 \times 2$
 $d = 10\text{m}$

b)
 $u = 0\text{ms}^{-1}$
 $v = ?$
 $a = 9.8\text{ms}^{-2}$
 $s = ?$
 $t = 2\text{s}$
 $v = u + at$
 $v = 0 + 9.8 \times 2$
 $v = 19.6\text{ms}^{-1}$

Example Two

During a game of rugby, a ball is kicked as shown below. The ball is kick with an initial speed of 18ms^{-1} at an angle of 65° to the ground.



- Calculate the initial horizontal component of the ball's velocity.
- Calculate the initial vertical component of the ball's velocity.
- By calculation show whether or not the ball crosses above the vertical bar on the posts.

Solution

a)

$$v_h = R \cos \Theta$$

$$v_h = 18 \cos 65$$

$$v_h = 7.6 \text{ms}^{-1}$$

b)

$$v_v = R \sin \Theta$$

$$v_v = 18 \sin 65$$

$$v_v = 16.3 \text{ms}^{-1}$$

c)

To solve this part we need to consider the horizontal and vertical motions separately.

Horizontal

$$v_h = 7.6 \text{ms}^{-1}$$

$$d = 21 \text{m}$$

$$t = ?$$

$$v_h = d / t$$

$$7.6 = 21 / t$$

$$t = 21 / 7.6$$

$$t = 2.8 \text{s}$$

Vertical (upwards is positive)

$$u = 16.3 \text{ms}^{-1}$$

$$v = ?$$

$$a = -9.8 \text{ms}^{-2}$$

$$s = ?$$

$$t = 2.8 \text{s}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = (16.3 \times 2.8) + (0.5 \times -9.8 \times (2.8)^2)$$

$$s = 45.6 + (-38.4)$$

$$s = 7.2 \text{m}$$

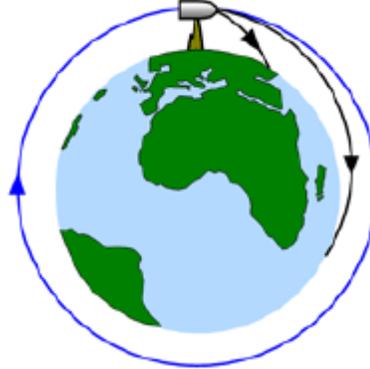
As this is 7.2m in the positive direction, the ball will clear the 3m high bar.

Satellite Motion

An object falling “freely” towards the surface of the Earth is said to be in “free fall”.

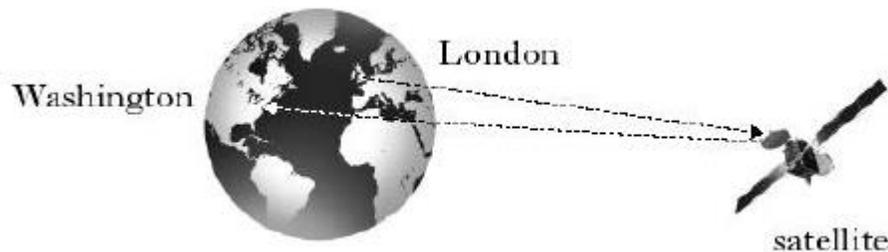
All objects accelerate towards the Earth with an acceleration of 9.8ms^{-2} (assuming no air resistance). An object in “free fall” inside a box would fall at the same rate as the box and would appear to be “weightless”.

If the falling object has a great enough horizontal velocity the object will never fall to the ground. The curvature of the Earth “falls” or curves away from the object at the same rate as the object. This idea was used by **Newton in his Thought Experiment**, and is the basis of satellite motion.



Satellites that orbit at **35,786km** above the Earth need an orbital (horizontal) velocity of 3139ms^{-1} to maintain their orbit. This will give the satellites a **period** (the time for one revolution) of **24hours** – which is the same time it takes the Earth to complete one revolution. This means these satellites will remain in a fixed position above the Earth’s surface – usually above the equator – making them very useful for communications and gathering weather data.

Satellites which have this type of orbit are known as **geostationary satellites**.



Gravitational Field Strength

The effect of gravitational field strength on an object decreases with height above the Earth. (An object would need to travel at $11,180\text{ms}^{-1}$ to escape the gravitational field on Earth. This is known as escape velocity.)

All masses in space will have associated gravitational field strengths. **The larger the mass the greater the gravitational field strength.** So an object like a star will have greater gravitational field strength than a planet which will have greater gravitational field strength than a moon.

Body	Gravitational Field Strength (Nkg^{-1})
Sun	274.13
Earth	9.81
Moon	1.62
Jupiter	25.95

Because all masses have associated gravitational fields, when **two masses** are in proximity to each other there will be a **force of attraction** between them. The Moon influences the tides on Earth.

Sir Isaac Newton proposed that the force of attraction between two objects could be calculated by the formula:

$$F = \frac{G m_1 m_2}{r^2}$$

where,

F is the force of attraction between the masses measured in Newtons

G is the gravitational constant of value $6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$

m_1 is the first mass in kilograms

m_2 is the second mass in kilograms

r is the distance between the masses in metres

This formula is known as **Newton's Law of Universal Gravitation.**

Example

Calculate the force of attraction between Planet A (mass $3.0 \times 10^{24}\text{kg}$) and Planet B (mass $8.9 \times 10^{24}\text{kg}$) which are separated by a distance of $3.0 \times 10^7\text{km}$.

Solution

$F = ?$

$G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$

$m_1 = 3.0 \times 10^{24}\text{kg}$

$m_2 = 8.9 \times 10^{24}\text{kg}$

$r = 3.0 \times 10^7\text{km} = 3.0 \times 10^{10}\text{m}$

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 3.0 \times 10^{24} \times 8.9 \times 10^{24}}{(3.0 \times 10^{10})^2}$$

$$F = 1.78 \times 10^{39} / 9 \times 10^{20}$$

$$F = 1.98 \times 10^{18}\text{N}$$

The table below contains the details of the Past Paper examples for this area of the course. Past Papers, and their solutions, are free to download from the SQA website.

Year	Section One	Section Two
2015	No examples	1 and 3
2016	5	1
2017	No examples	5a)ii
2018	5	1
2019	2 and 3	4

SPECIAL RELATIVITY

The earliest analyses of motion were based on the principle that “motion is relative to a fixed reference point” e.g.; the start of a race. Newton and Galileo developed this theory and assumed that laws established in the laboratory held true universally and were not dependent on where they were measured or how the observer might be in motion while making the observation. This theory became known as **Galilean invariance**.

In the 19th century a Scottish Physicist, Maxwell, began to doubt this theory. Maxwell believed that the **speed of light in a vacuum had a fixed value** which could be calculated from:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

where,

c is the speed of light in a vacuum

ϵ_0 is the permittivity of free space

μ_0 is the permeability of free space

Einstein took this result and worked it through to its conclusion based on two postulates:

- The speed of light is a constant value in every frame of reference
- Galilean invariance holds true for all frames of reference.

Einstein worked out that a stationary observer watching an object moving at a speed close to the speed of light would record a longer time than an observer moving in the object. This is known as **time dilation**. Time dilation can be calculated using the following equation:

$$t' = \gamma t$$

where,

t' is the observed time for the stationary observer

t is the time experienced in the moving object

γ is the Lorentz factor

Higher – Our Dynamic Universe – Summary Notes

He also worked out that a stationary observer watching an object moving at a speed close to the speed of light would record a shorter length than an observer in the object. This is known as **length contraction**. Length contraction can be calculated using the following equation:

$$l' = l / \gamma$$

where,

l' is the length recorded by the stationary observer

l is the length recorded in the moving object

γ is the Lorentz factor

Note the **Lorentz factor** can be calculated using the following equation:

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

Experimental verification of time dilation and length contraction has been achieved from muon detection at the surface of the Earth and time measurements in airborne clocks.

(Both of these verifications are worthwhile background reading to supplement the course content.)

Example

A spacecraft travels at a constant speed of $0.8c$ relative to Earth. A clock on the spacecraft records a time of 12 hours. Calculate the time shown on a clock on Earth that records the same flight.

Solution

Before using the time dilation equation you must calculate the Lorentz factor.

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

Step One

Calculate the bottom line.

$$\sqrt{1 - (v/c)^2}$$

$$\sqrt{1 - (0.8c/c)^2}$$

$$\sqrt{1 - (0.8)^2}$$

$$\sqrt{1 - 0.64}$$

$$\sqrt{0.36}$$

$$0.6$$

Step Two

Calculate the Lorentz factor.

$$\gamma = 1 / 0.6$$

$$\gamma = 1.67$$

Step Three

Calculate the time observed by the stationary observer.

$$t' = t\gamma$$

$$t' = 12 \times 1.67$$

$$t' = 20 \text{ hours}$$

Example

A stationary observer on Earth views a spacecraft travelling at $0.9c$. Calculate the length of the spacecraft at rest if its observed length is 60m as it passes by.

Solution

Before using the length contraction equation you must calculate the Lorentz factor.

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

Step One

Calculate the bottom line.

$$\sqrt{1 - (v/c)^2}$$

$$\sqrt{1 - (0.9c/c)^2}$$

$$\sqrt{1 - (0.9)^2}$$

$$\sqrt{1 - 0.81}$$

$$\sqrt{0.19}$$

$$0.44$$

Step Two

Calculate the Lorentz factor.

$$\gamma = 1 / 0.44$$

$$\gamma = 2.3$$

Step Three

Calculate the length of the space craft

$$l' = l / \gamma$$

$$60 = l / 2.3$$

$$l = 138\text{m}$$

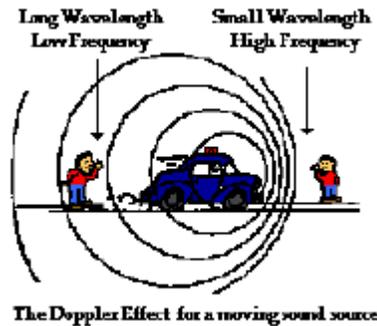
The table below contains the details of the Past Paper examples for this area of the course. Past Papers, and their solutions, are free to download from the SQA website.

Year	Section One	Section Two
2015	7	No examples
2016	No examples	4
2017	4	No examples
2018	6 and 7	No examples
2019	8	7 c)ii)

THE EXPANDING UNIVERSE

The Doppler Effect

The Doppler Effect is the change in frequency observed when a source of sound waves is moving relative to an observer. When the source of sound waves moves towards the observer, more waves are received per second and the frequency heard is increased. Similarly, as the source of sound waves moves away from the observer fewer waves are received each second and the frequency heard decreases.



The Doppler Effect can be observed from the sirens of moving vehicles as shown above or from a train “whistling” as it goes under a bridge. The Doppler Effect has many applications including speed measurement in RADAR guns and echocardiograms.

In general, **for a stationary observer**, the frequency that is heard can be calculated from:

$$f_{\text{obs}} = f_s \frac{v}{(v \pm v_s)}$$

where,

f_{obs} is the frequency heard by the stationary observer in Hertz

f_s is the frequency of the source of the sound in Hertz

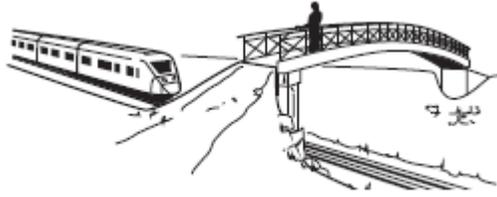
v is the velocity of the wave in ms^{-1}

v_s is the velocity of the source in ms^{-1}

When applying the equation the (-) should be selected for an emitter **moving towards** the observer and the (+) should be selected for an emitter **moving away** from the observer.

Example

A train is approaching a bridge with a constant speed of 12.0ms^{-1} . The train sounds its horn which has a frequency of 270Hz . Calculate the frequency heard by a stationary person standing on the bridge if the speed of sound in air is 340ms^{-1} .



Solution

As the train is approaching the observer use the (-) in the Doppler Effect equation:

$$f_{\text{obs}} = f_s \frac{v}{(v - v_s)}$$

$f_{\text{obs}} = ?$
 $f_s = 270\text{Hz}$
 $v = 340\text{ms}^{-1}$
 $v_s = 12.0\text{ms}^{-1}$

$f_{\text{obs}} = f_s \times v / (v - v_s)$
 $f_{\text{obs}} = 270 \times 340 / (340 - 12)$
 $f_{\text{obs}} = 270 \times 340 / 328$
 $f_{\text{obs}} = 280\text{Hz}$

The **Doppler Effect** causes similar shifts in **wavelengths of light**.

The light from objects moving away from us is shifted to longer wavelengths – this is known as **red shift**.

The light from objects moving towards us is shifted to shorter wavelengths – this is known as **blue shift**.

When **spectral analysis of stars** is carried out to identify the chemical elements that they contain the Doppler Effect must be taken into account. In doing this we can work out the **motion of the star** relative to us as well as its chemical composition.

The physical quantity red shift, symbol z , is the change in wavelength divided by the emitted wavelength.

$$z = (\lambda_{\text{observed}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

This calculation will always give a positive number for stars moving away from us and a negative number for stars moving towards us.

Higher – Our Dynamic Universe – Summary Notes

It should be noted that the Doppler Effect equation for sound cannot be used for light travelling at velocities greater than $0.1c$. However, at slower velocities red shift can also be found from the equation:

$$z = v / c$$

where,

z is the red shift

v is the velocity of the object being observed

c is the speed of light in a vacuum ($3 \times 10^8 \text{ms}^{-1}$)

Example One

A galaxy is moving away from Earth at a velocity of $2.4 \times 10^7 \text{ms}^{-1}$. What is the red shift value for this galaxy?

$$z = ?$$

$$v = 2.4 \times 10^7 \text{ms}^{-1}$$

$$c = 3.0 \times 10^8 \text{ms}^{-1}$$

$$z = v / c$$

$$z = 2.4 \times 10^7 / 3.0 \times 10^8$$

$$z = 0.08$$

Example Two

The red shift observed from a star was 0.04. If the rest wavelength of a spectral line is known to be 550nm, calculate the observed wavelength of this spectral line.

$$z = 0.04$$

$$\lambda_{\text{observed}} = ?$$

$$\lambda_{\text{rest}} = 540\text{nm}$$

$$z = (\lambda_{\text{observed}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$$0.04 = (\lambda_{\text{observed}} - 550) / 550$$

$$0.04 \times 550 = (\lambda_{\text{observed}} - 550)$$

$$22 = \lambda_{\text{observed}} - 550$$

$$\lambda_{\text{observed}} = 572\text{nm}$$

Hubble's Law and the Expanding Universe

Hubble's Law, which is shown below, is the relationship between the recession velocity of a galaxy and its distance from us.

$$v = H_0 d$$

The **Hubble constant** in the above equation has a value, in SI units, of $2.34 \times 10^{-18} \text{s}^{-1}$.

This law implies that all galaxies (stars) are moving away from us and that the galaxies (stars) that are further away are moving faster than those that are closer to us.

Hubble's Law also leads to an estimate of the **age of the Universe as 13 billion years**.

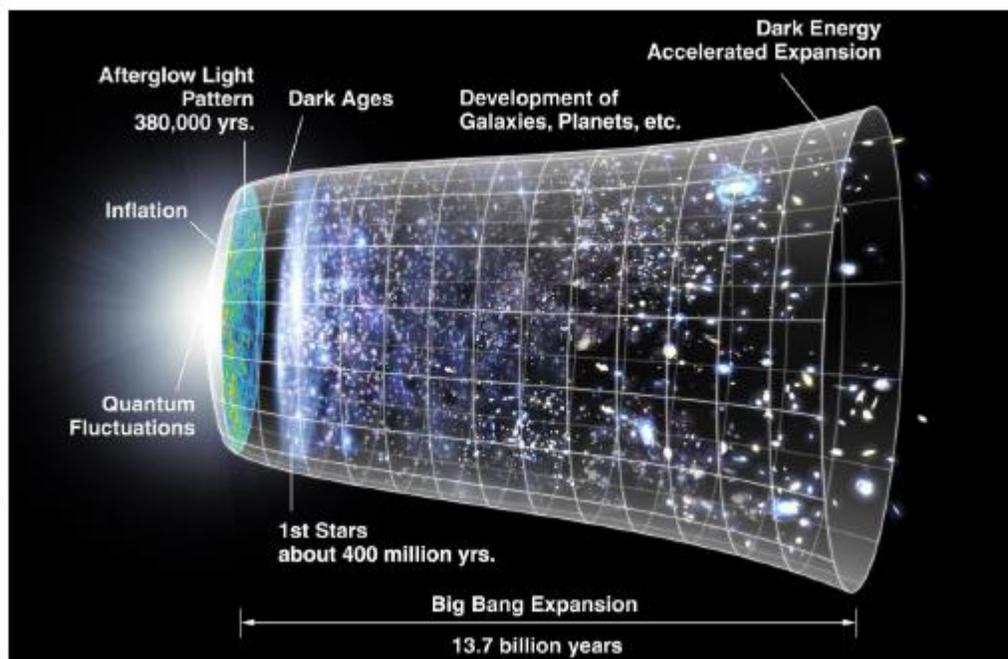
As Hubble's Law tells us the Universe is expanding is it possible to predict the rate of expansion?

The evidence that has been gathered on the masses and velocities of galaxies has led to an astounding conclusion – a new type of matter must exist. This has been named **dark matter**. Not only does this dark matter exist but it makes up more of the Universe than “normal” matter.

Current estimates suggest that “normal” matter makes up 5% of the Universe and dark matter makes up 25% of the Universe.

What makes up the other 70%?

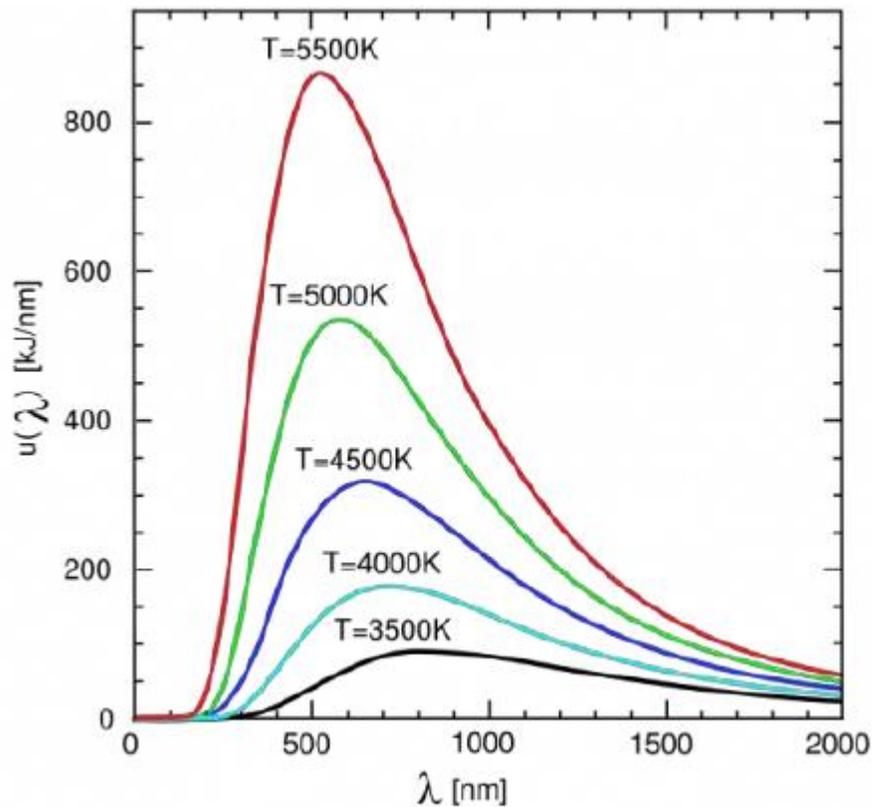
The remainder of the Universe is thought to consist of **dark energy**. This dark energy is believed to be able to overcome gravitational attractions and is causing the Universe to have an **accelerating expansion rate**.



The above image gives a timeline for the evolution of the Universe. The far left represents the earliest moments of the Universe and the right hand side depicts the accelerating expansion caused by dark energy.

Stellar Temperatures

Stellar objects emit radiation over a wide range of wavelengths and therefore over a wide range of energies. (This deduction comes from the equations $E = hf$ and $v = f\lambda$. Both of which are covered in more detail elsewhere in this course.) Although the distribution of energy is spread over a wide range of wavelengths, each object emitting radiation has a **peak wavelength which depends on its temperature** (thermal energy).



In the above graph the **peak wavelength is shorter for hotter stellar objects** than for cooler stellar objects. Also, hotter objects emit more radiation per unit surface area at all wavelengths than cooler objects. This means the area under the curve will be greater for hotter objects than for cooler objects. The curves shown are the typical shape for a “blackbody radiator”.

So we can study the temperature of stellar objects from Earth by measuring the intensities of the various wavelengths of light that they emit. This has led to the classification of stars based on their surface temperature. Stars are classified using the letters O, B, A, E, G, K and M and further background reading on this is available using the Harvard spectral classification.

The Big Bang Theory

The term Big Bang was used by astronomer Fred Hoyle who disagreed with the idea of a continually expanding Universe. But the temperature measurements that have been taken from space since 1965 support the expanding Universe idea and therefore support the Big Bang Theory.

These temperature measurements have been obtained from research into Cosmic Microwave Background Radiation (CMBR). If CMBR exists there should be two characteristics that can be detected:

- CMBR should be uniformly distributed across the Universe.
- The intensity curve for CMBR should have the shape of a “blackbody radiator” with a predictable peak wavelength.

The Cosmic Background Explorer (COBE) satellite has provided evidence of the temperature of the Universe which supports the temperature predicted by the expansion and subsequent cooling of the Universe following the Big Bang.

Other evidence that supports the Big Bang is the even distribution of matter in the Universe and the abundance of light elements such as Helium throughout the Universe.

Of course, this theory is very contentious and will continue to be debated... by Sheldon and lots of others!

The table below contains the details of the Past Paper examples for this area of the course. Past Papers, and their solutions, are free to download from the SQA website.

Year	Section One	Section Two
2015	8	4b)
2016	6 and 7	5
2017	5, 6 and 7	1b) and 5b)
2018	No examples	5 and 10c)
2019	9 and 10	5 a) and 6 a)

DATA SHEET

COMMON PHYSICAL QUANTITIES

Quantity	Symbol	Value	Quantity	Symbol	Value
Speed of light in vacuum	c	$3.00 \times 10^8 \text{ m s}^{-1}$	Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Magnitude of the charge on an electron	e	$1.60 \times 10^{-19} \text{ C}$	Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Universal Constant of Gravitation	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Gravitational acceleration on Earth	g	9.8 m s^{-2}	Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Hubble's constant	H_0	$2.3 \times 10^{-18} \text{ s}^{-1}$			

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	Refractive index	Substance	Refractive index
Diamond	2.42	Water	1.33
Crown glass	1.50	Air	1.00

SPECTRAL LINES

Element	Wavelength/nm	Colour	Element	Wavelength/nm	Colour
Hydrogen	656	Red	Cadmium	644	Red
	486	Blue-green		509	Green
	434	Blue-violet		480	Blue
	410	Violet	Lasers		
	397	Ultraviolet	Element	Wavelength/nm	Colour
	389	Ultraviolet	Carbon dioxide	9550 } 10590 }	Infrared
Sodium	589	Yellow	Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	Density/kg m ⁻³	Melting Point/K	Boiling Point/K
Aluminium	2.70×10^3	933	2623
Copper	8.96×10^3	1357	2853
Ice	9.20×10^2	273	...
Sea Water	1.02×10^3	264	377
Water	1.00×10^3	273	373
Air	1.29
Hydrogen	9.0×10^{-2}	14	20

The gas densities refer to a temperature of 273 K and a pressure of $1.01 \times 10^5 \text{ Pa}$.

Higher – Our Dynamic Universe – Summary Notes

$d = \bar{v}t$	$E_w = QV$	$V_{peak} = \sqrt{2}V_{rms}$
$s = \bar{v}t$	$E = mc^2$	$I_{peak} = \sqrt{2}I_{rms}$
$v = u + at$	$E = hf$	$Q = It$
$s = ut + \frac{1}{2}at^2$	$E_K = hf - hf_0$	$V = IR$
$v^2 = u^2 + 2as$	$E_2 - E_1 = hf$	$P = IV = I^2R = \frac{V^2}{R}$
$s = \frac{1}{2}(u + v)t$	$T = \frac{1}{f}$	$R_T = R_1 + R_2 + \dots$
$W = mg$	$v = f\lambda$	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$
$F = ma$	$d \sin \theta = m\lambda$	$E = V + Ir$
$E_w = Fd$	$n = \frac{\sin \theta_1}{\sin \theta_2}$	$V_1 = \left(\frac{R_1}{R_1 + R_2} \right) V_s$
$E_p = mgh$	$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$	$\frac{V_1}{V_2} = \frac{R_1}{R_2}$
$E_K = \frac{1}{2}mv^2$	$\sin \theta_c = \frac{1}{n}$	$C = \frac{Q}{V}$
$P = \frac{E}{t}$	$I = \frac{k}{d^2}$	$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2} \frac{Q^2}{C}$
$p = mv$	$I = \frac{P}{A}$	
$Ft = mv - mu$	path difference = $m\lambda$ or $\left(m + \frac{1}{2}\right)\lambda$ where $m = 0, 1, 2, \dots$	
$F = G \frac{m_1 m_2}{r^2}$	random uncertainty = $\frac{\text{max. value} - \text{min. value}}{\text{number of values}}$	
$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$		
$l = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$		
$f_o = f_s \left(\frac{v}{v \pm v_s} \right)$		
$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$		
$z = \frac{v}{c}$		
$v = H_0 d$		

