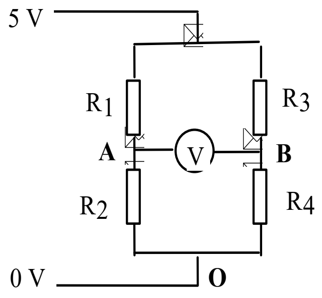
HIGHER CIRCUIT THEORY

Wheatstone Bridge Circuit

Any method of measuring resistance using an ammeter or voltmeter necessarily involves some error unless the resistances of the meters themselves are taken into account. The use of digital voltmeters largely overcomes this as they tend to have very high resistances. Further error may be introduced if the meter is not correctly calibrated. The only situation where neither of these errors matter is if the meter reading is zero. The Wheatstone bridge circuit is one such example of using a meter as a null deflection indicator.

Bridge Circuit

If $V_{OA} = V_{OB}$ there is no p.d. between A and B hence no current flows. The potentials at A and at B depend on the ratio of the resistors that make up each of the two voltage dividers. The voltmeter forms a 'bridge' between the two voltage dividers to make up a bridge circuit.



Balanced Bridge

No potential difference will exist across AB when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

The bridge is balanced when the voltmeter or galvanometer (milliammeter) reads zero.

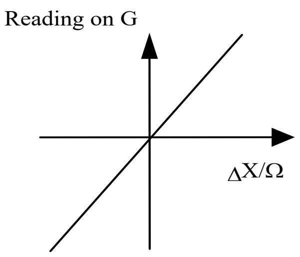
Alternative 'diamond'
Representation with
galvanometer

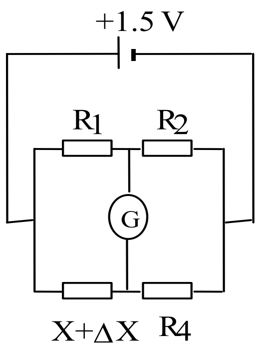
Unbalanced Bridge

If the bridge is initially balanced, and the resistor × is altered by a small amount ☚× then the out of balance p.d. (reading on G) is directly proportional to the change in resistance, provided the change is small.

Hence for small ☚×,

reading on G } ☚×

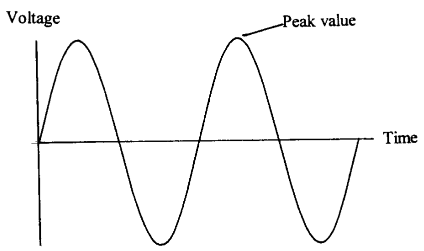




ALTERNATING CURRENT AND VOLTAGE

Peak and r.m.s. values

The graph of a typical alternating voltage is shown below.

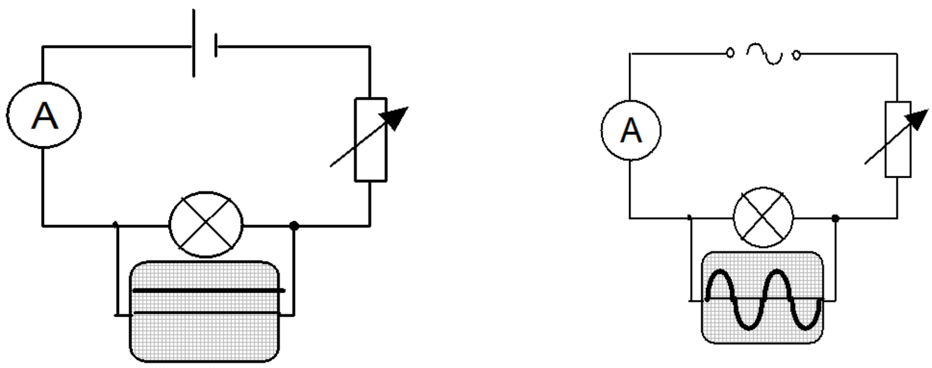


The maximum voltage is called the **peak value**.
From the graph it is obvious that the peak value would not be a very accurate measure of the voltage available from an alternating supply.

In practice the value quoted is the **root mean square** (r.m.s.) voltage.

The r.m.s. value of an alternating voltage or current is defined as being equal to the value of the direct voltage or current which gives rise to the same heating effect (same power output).

Consider the following two circuits which contain identical lamps.



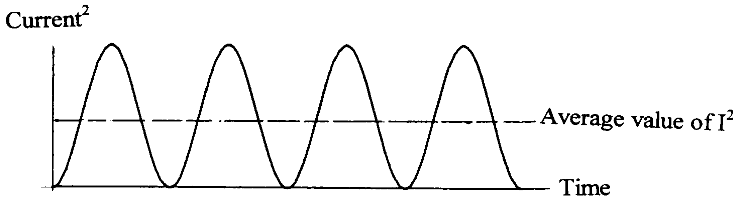
The variable resistors are altered until the lamps are of equal brightness. As a result the direct current has the same value as the effective alternating current (i.e. the lamps have the same power output). Both voltages are measured using an oscilloscope giving the voltage equation below. Also, since V=IR applies to the r.m.s. valves and to the peak values a similar equation for currents can be deduced.

$$V_{r.m.s.} = \frac{1}{\sqrt{2}} V_{peak} \quad \text{and} \quad I_{r.m.s.} = \frac{1}{\sqrt{2}} I_{peak}$$

Note: a moving coil a.c. meter is calibrated to give r.m.s. values.

Graphical method to derive relationship between peak and r.m.s. values of alternating current

The power produced by a current I in a resistor R is given by $I^2 R$. A graph of I^2 against t for an alternating current is shown below. A similar method can be used for voltage.



The average value of I^2 is $\frac{I^2_{Peak}}{2}$

An identical heating effect (power output) for a d.c. supply = $I^2_{r.m.s} R$
[since I (d.c.) = $I_{r.m.s.}$]

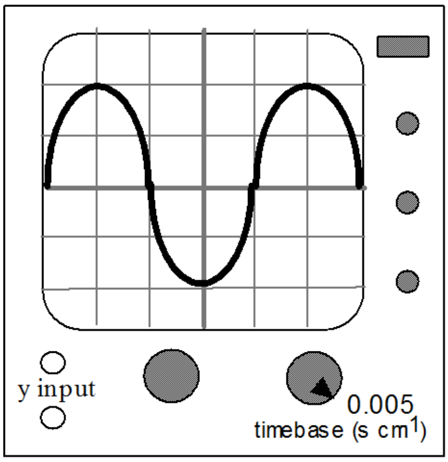
Average power output for a.c. = $\frac{I^2_{Peak}}{2} R$

$I^2_{r.m.s} R = \frac{I^2_{Peak}}{2} R$ hence $\oplus I^2_{r.m.s.} = \oplus \frac{I^2_{Peak}}{2}$ giving $I_{r.m.s.} = \frac{I_{Peak}}{\oplus 2}$

Frequency of a.c.

To describe the domestic supply voltage fully, we would have to include the frequency i.e. 230 V 50 Hz.

An oscilloscope can be used to find the frequency of an a.c. supply as shown below.



Time base = 0.005 s cm^{-1}

Wavelength = 4 cm

Time to produce one wave = 4 × 0.005
= 0.02 s

$$\text{Frequency} = \frac{1}{\text{time to produce one wave}}$$

$$= \frac{1}{0.02} = 50 \text{ Hz}$$

Mains supply

The mains supply is usually quoted as 230 V a.c. This is of course 230 $V_{r.m.s.}$ The peak voltage rises to approximately 325 V. Insulation must be provided to withstand this peak voltage.

Example

A transformer is labelled with a primary of 230 $V_{r.m.s.}$ and secondary of 12 $V_{r.m.s.}$ What is the peak voltage which would occur in the secondary?

$$V_{peak} = \sqrt{2} \times V_{r.m.s.}$$

$$V_{peak} = 1.41 \times 12$$

$$V_{peak} = 17.0\ V$$

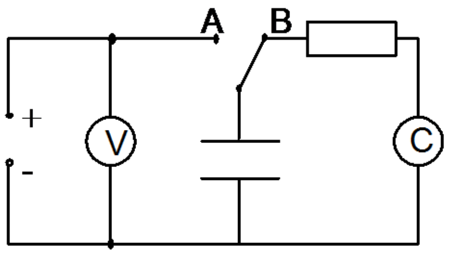
CAPACITANCE

The ability of a component to store charge is known as **capacitance.**
A device designed to store charge is called a **capacitor.**

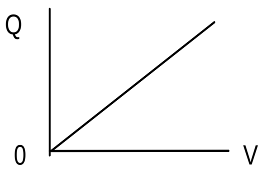
A typical capacitor consists of two conducting layers separated by an insulator.

Circuit symbol ωδ

Relationship between charge and p.d.



The capacitor is charged to a chosen voltage by setting the switch to A. The charge stored can be measured directly by discharging through the coulomb meter with the switch set to B. In this way pairs of readings of voltage and charge are obtained.



Charge is directly proportional to voltage.

$$\frac{Q}{V} = \text{constant}$$

For any capacitor the ratio Q/V is a constant and is called the capacitance.

farad (F) —— capacitance = $\dfrac{\text{charge}}{\text{voltage}}$ —— charge: coulombs (C); voltage: volts (V)

The farad is too large a unit for practical purposes.
In practice the micro farad (μF) = 1×10^{-6} F and the nano farad (nF) = 1×10^{-9} F are used.

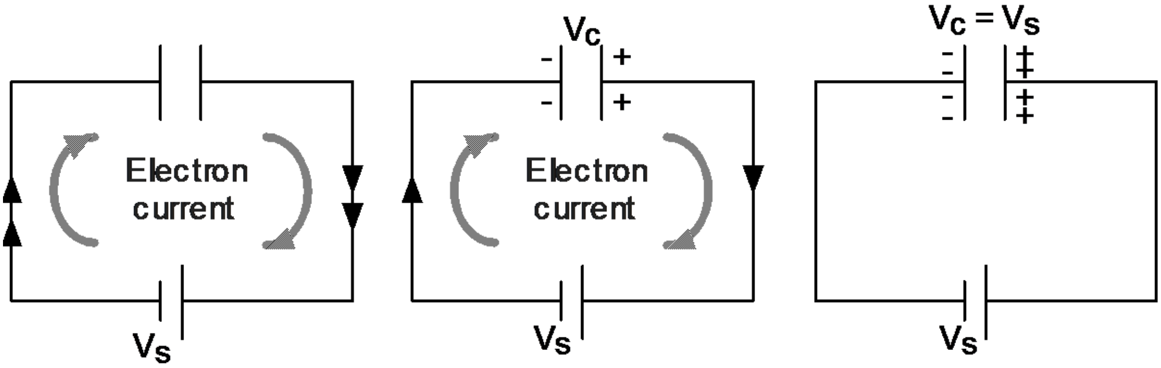
Example
A capacitor stores 4×10^{-4} C of charge when the potential difference across it is 100 V. What is the capacitance ?

$$C = \frac{Q}{V} = \frac{4 \times 10^{-6}}{100}$$

$= 4\ \mu F$

Energy Stored in a Capacitor

A charged capacitor can be used to light a bulb for a short time, therefore the capacitor must contain a store of energy. The charging of a parallel plate capacitor is considered below.

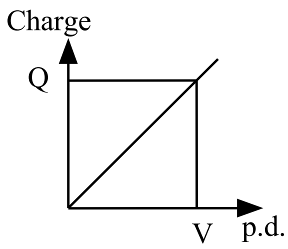


There is an initial surge of electrons from the negative terminal of the cell onto one of the plates (and electrons out of the other plate towards the +ve terminal of the cell).

Once some charge is on the plate it will repel more charge and so the current decreases. In order to further charge the capacitor the electrons must be supplied with enough energy to overcome the potential difference across the plates i.e. work is done in charging the capacitor.

Eventually the current ceases to flow. This is when the p.d. across the plates of the capacitor is equal to the supply voltage.

For a given capacitor the p.d. across the plates is directly proportional to the charge stored
Consider a capacitor being charged to a p.d. of V and holding a charge Q.



The energy stored in the capacitor is given by the area under graph

Area under graph $= \frac{1}{2}$ Q x V

Energy stored $= \frac{1}{2}$ Q x V

If the voltage across the capacitor was constant work done = Q x V, but since V is varying, the work done = area under graph.

Q = C × V and substituting for Q and V in our equation for energy gives:

$$\text{Energy stored in a capacitor} = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Example

A 40③F capacitor is fully charged using a 50 V supply. How much energy is stored?

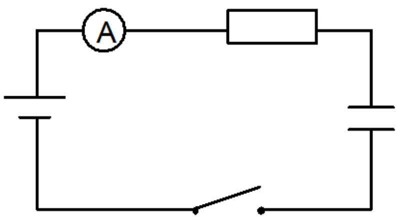
$$Energy = \frac{1}{2}CV^2 = \frac{1}{2} \times 40 \times 10^{6} \times 2500$$

$$= 5 \times 10^{-2}\ J$$

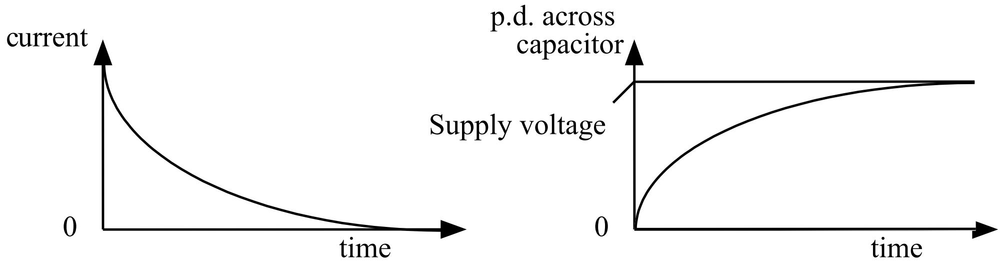
Capacitance in a d.c. Circuit

Charging

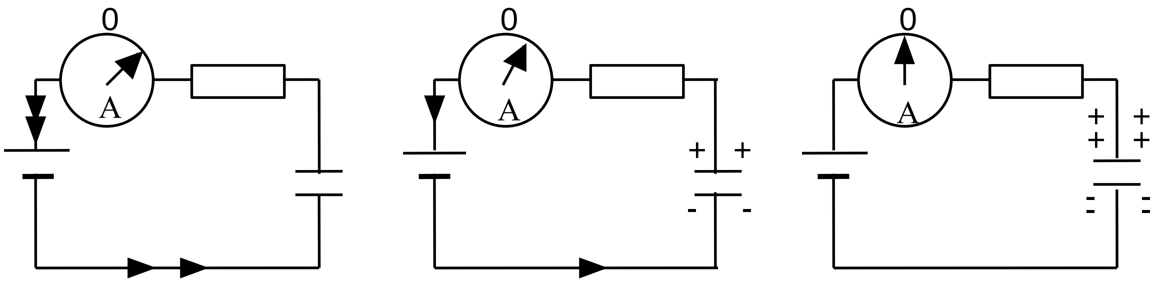
Consider the following circuit:-



When the switch is closed the current flowing in the circuit and the voltage across the capacitor behave as shown in the graphs below.



Consider the circuit at three different times.



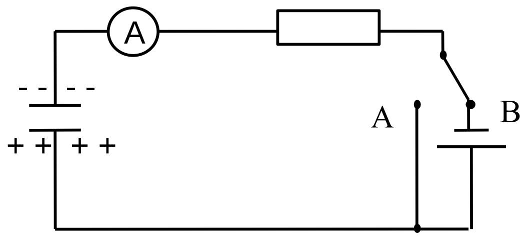
As soon as the switch is closed there is no charge on the capacitor the current is limited only by the resistance in the circuit and can be found using Ohm's law.

As the capacitor charges a p.d. develops across the plates which opposes the p.d. of the cell as a result the supply current decreases.

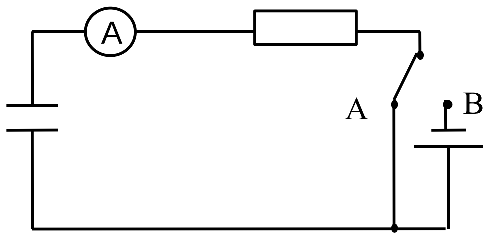
The capacitor becomes fully charged and the p.d. across the plates is equal and opposite to that across the cell and the charging current becomes zero.

Discharging

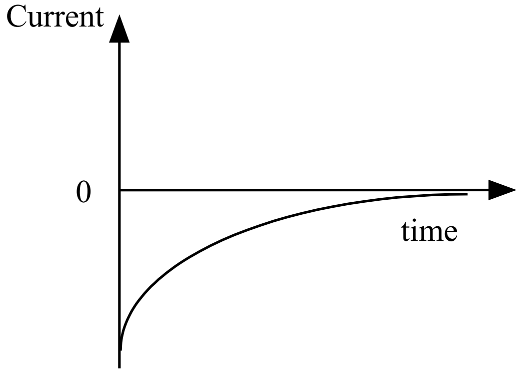
Consider this circuit when the capacitor is fully charged, switch to position B

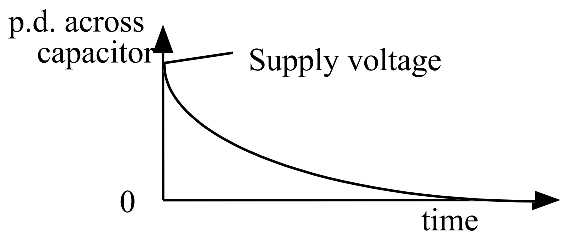


If the cell is taken out of the circuit and the switch is set to A, the capacitor will discharge



While the capacitor is **discharging** the current flowing in the circuit and the voltage across the capacitor behaves as shown in the graphs below.

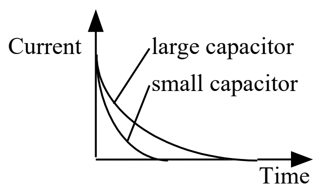


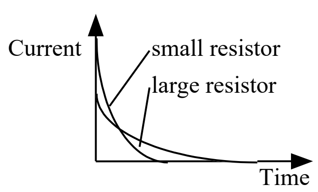


Although the current/time graph has the same shape as that during charging the currents in each case are flowing in opposite directions. The discharging current decreases because the p.d. across the plates decreases as charge leaves them.

Factors affecting the rate of charge/discharge of a capacitor

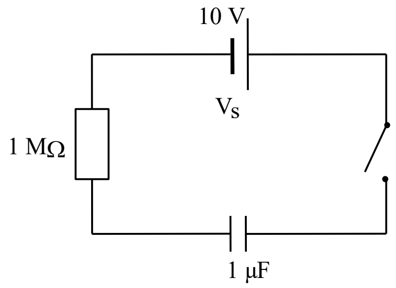
- When a capacitor is charged to a given voltage the time taken depends on the value of the capacitor. The larger the capacitor the longer the charging time, since a larger capacitor requires more charge to raise it to the same p.d. as a smaller capacitor as $V = \frac{Q}{C}$
- When a capacitor is charged to a given voltage the time taken depends on the value of the resistance in the circuit. The larger the resistance the smaller the initial charging current, hence the longer it takes to charge the capacitor as Q = It





(The area under this I/t graph = charge. Both curves will have the same area since Q is the same for both.)

Example
The switch in the following circuit is closed at time t = 0



Immediately after closing the switch what is:
(a) the charge on C
(b) the p.d. across C
(c) the p.d. across R
(d) the current through R.

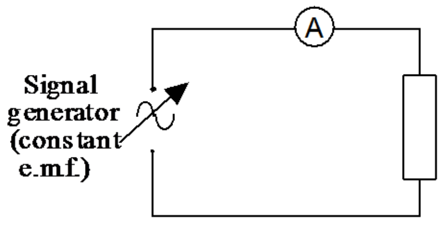
When the capacitor is fully charged what is:
(e) the p.d. across the capacitor
(f) the charge stored.

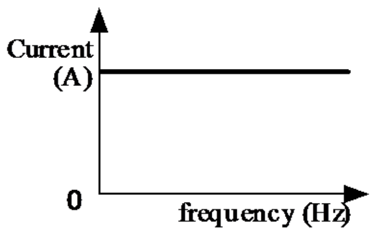
(a) *Initial charge on capacitor is zero.*
(b) *Initial p.d . is zero since charge is zero.*
(c) *p.d. is 10 V =* $V_s - V_c = 10 - 0 = 10$ *V*
(d) $\frac{I}{R} = V = \frac{10}{10^6} = 10^3$ *A*
(e) *Final p.d. across the capacitor equals the supply voltage = 10 V.*
(f) $Q = CV = 2 \times 10^{-6} \times 10 = 2 \times 10^{-5}$ *C.*

Resistors and Capacitors in a.c. Circuits

Frequency response of resistor

The following circuit is used to investigate the relationship between current and frequency in a resistive circuit.

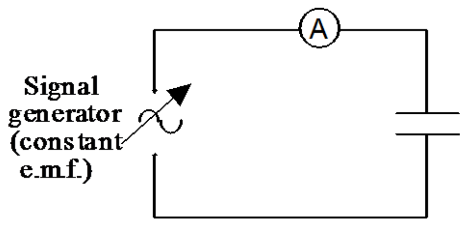


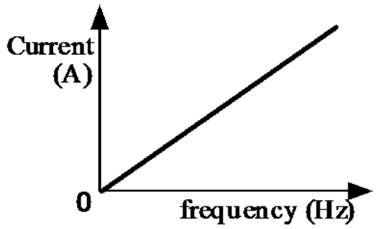


The results show that the current flowing through a resistor is independent of the frequency of the supply.

Frequency response of capacitor

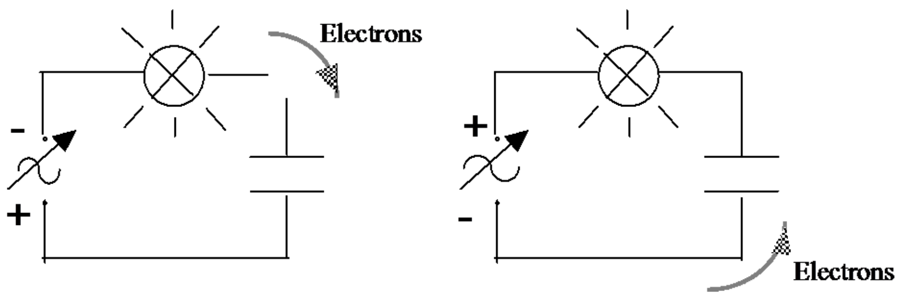
The following circuit is used to investigate the relationship between current and frequency in a capacitive circuit.





The results show that the current is directly proportional to the frequency of the supply.

To understand the relationship between the current and frequency consider the two halves of the a.c. cycle.

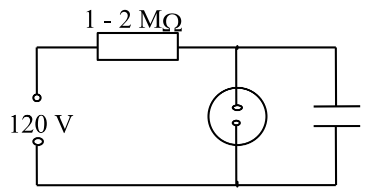


The electrons move back and forth around the circuit passing through the lamp and charging the capacitor one way and then the other (the electrons do not pass through the capacitor). The higher the frequency the less time there is for charge to build up on the plates of the capacitor and oppose further charges from flowing in the circuit More charge is transferred in one second so the current is larger.

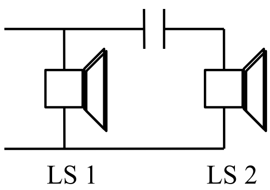
Applications of Capacitors (for background interest)

Blocking capacitor
A capacitor will stop the flow of a steady d.c. current . This is made use of in the a.c./d.c. switch in an oscilloscope. In the a.c. position a series capacitor is switched in allowing passage of a.c. components of the signal, but blocking any steady d.c. signals.

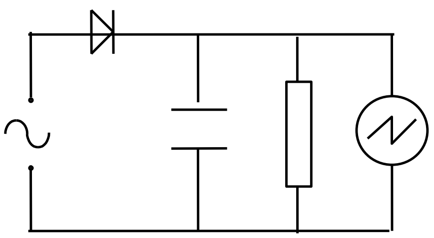
Flashing indicators
A low value capacitor is charged through a resistor until it acquires sufficient voltage to fire a neon lamp. The neon lamp lights when the p.d. reaches 100 V. The capacitor is quickly discharged and the lamp goes out when the p.d. falls below 80V.

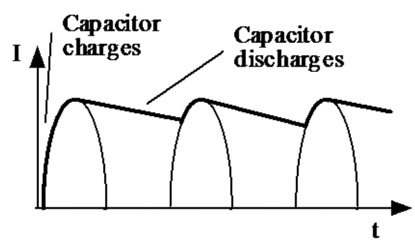


Crossover networks in loudspeakers
In a typical crossover network in low cost loudspeaker systems, the high frequencies are routed to LS-2 by the capacitor.



Smoothing
The capacitor in this simple rectifier circuit is storing charge during the half cycle that the diode conducts. This charge is given up during the half cycle that the diode does not conduct. This helps to smooth out the waveform.





Capacitor as a transducer
A parallel plate capacitor can be used to convert mechanical movements or vibration of one of its plates into changes in voltage. This idea forms the basis of many measuring systems, e.g. by allowing a force to compress the plates we have a pressure transducer.

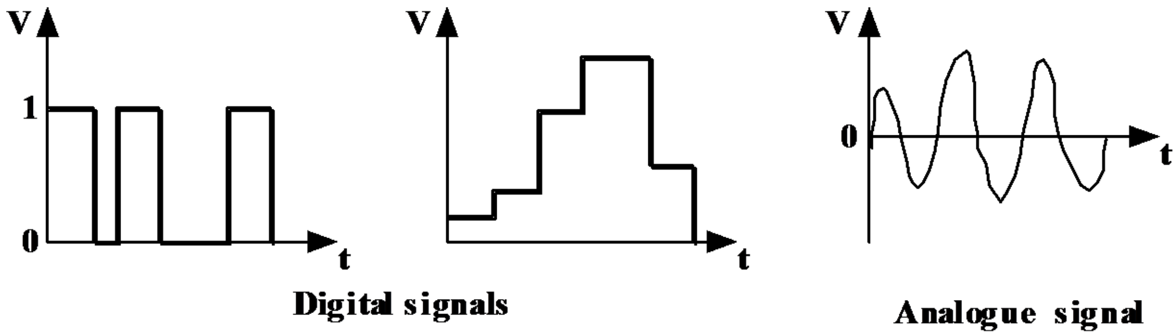
ANALOGUE ELECTRONICS

Analogue and Digital Signals

An analogue system transmits information in the form of a continuously varying signal.
A digital system has the information broken up into a series of discrete values or steps.

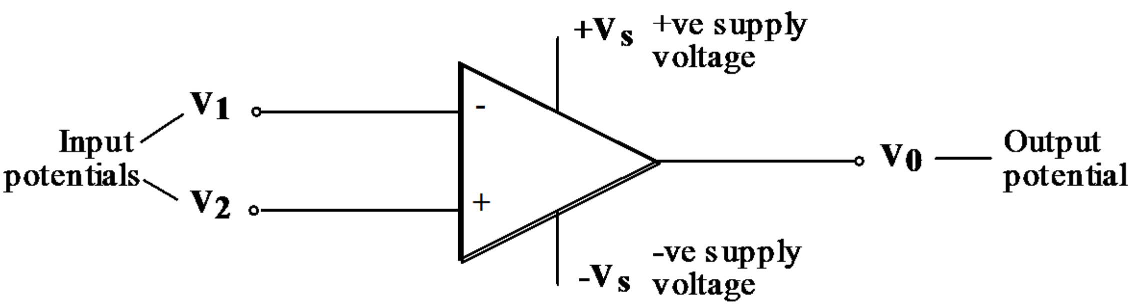
A simple example showing these definitions would be analogue and digital watches.

- an analogue watch has a set of numbers in a circle and hands that point to them. The hands sweep round the face in a continuous way.
- a digital watch has a series of numbers that are displayed, the numbers changing at the end of each second, minute or hour.



The Op-Amp (Operational Amplifier)

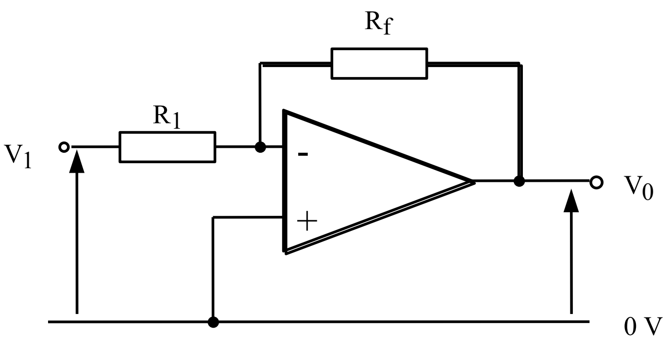
As with any amplifier, the operational amplifier (op-amp) will change the size of an input electrical signal. A diagram of the basic set-up of an op-amp is shown below.



The op-amp has two separate inputs - an inverting input ("-" terminal) and a non-inverting input ("+" terminal). The amplifier must have an energy supply and this is provided by the supply voltages +Vs and -Vs across the amplifier. Often the power supply voltages are not shown on circuit diagrams of op-amps.

The Inverting Amplifier

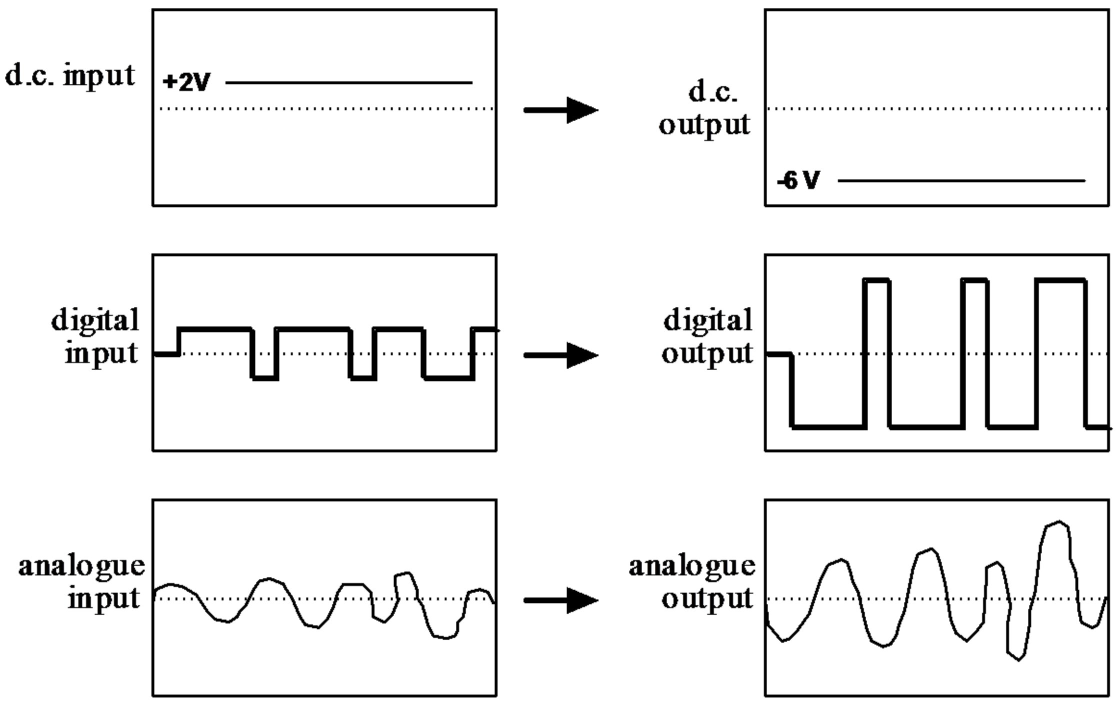
If an amplifier is set up in the configuration shown below, it is said to be in the **inverting mode.**



The input potential, V_1, is applied to the **inverting input** (-ve input). The non-inverting input (+ve input) is connected straight to "ground", 0 V.

There is also a resistor, $\mathbf{R_f}$, (feedback resistor) connected between the output and the inverting input. This feedback resistor reduces the overall gain of the amplifier and allows the gain to be stabilised controlled.

When the input voltage signal to the op-amp, d.c or a.c, is compared to the output voltage signal it is found that the sense or 'sign' of the output is opposite to that of the input - the voltage has been **inverted**, hence the reason why this circuit is called the "inverting mode". Some examples of this are given below:



Inverting Mode Gain Equation

The **ideal** op-amp fulfils two conditions:

- no current flows into the op-amp. Its input resistance is infinite.
- there is no potential difference between the inverting and non-inverting inputs.

The following inverting mode gain equation can be verified by experiment.

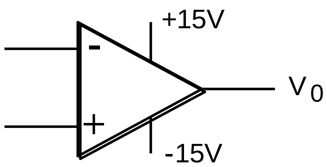
$$\frac{V_0}{V_1} = -\frac{R_f}{R_1} \quad \text{or} \quad V_0 = -\frac{R_f}{R_1} V_1$$

Saturation

From the inverting mode gain equation, it would seem that by inserting any pair of resistors R_1 and R_f in the inverting amplifier circuit it is possible to provide any gain required. It could therefore be possible to produce either very low or very high output voltages from any input voltage.

This is **not** possible. The output of an op-amp circuit is limited by the size of the power supply used (conservation of energy). In theory, the maximum output voltage possible would be equal to the supply voltage. (However, in reality it is limited to approximately 85% of the supply voltage.)

When the maximum output voltage has been reached, the amplifier is said to be **saturated.**



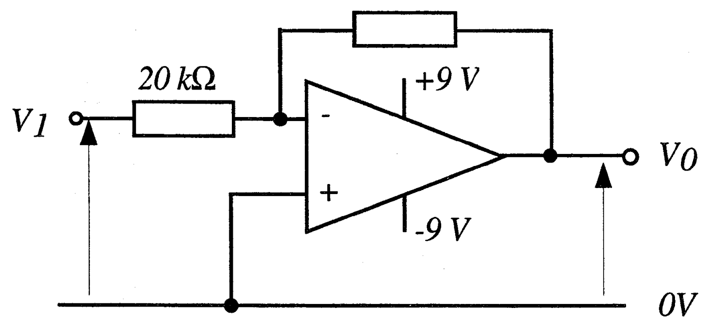
In theory : **$-15V \leq V_0 \leq +15$ V**

[In practice : **$-13V \leq V_0 \leq +13$ V** (13V ≈ 85% of 15 V)]

Example

An inverting mode operational amplifier is set up as shown below.

(a) *If V_1 is set at +0.8 V, calculate the output voltage V_0.*
(b) *If an a.c signal of peak voltage 1.5 V is applied to V_1, sketch the input voltage, V_1, and the output voltage, V_0.*



(a) $R_f = 100\,k\Omega,\ R_1 = 20\,k\Omega,\ V_1 = +0.8\ V,\ V_0 = ?$

$$V_0 = -\frac{R_f}{R_1} V_1 = -\frac{100}{20} \times 0.8$$

$$V_0 = -4.0\ V$$

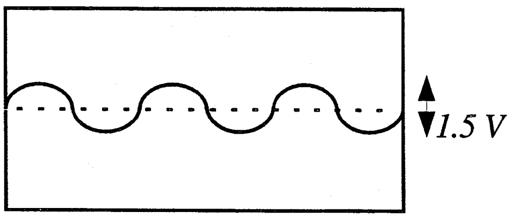
(b) *Input trace has a peak of 1.5 V, therefore the peak value of the output trace is given by :*

$R_f = 100\,k\Omega,\ R_1 = 20\,k\Omega,\ V_1 = +1.5\ V,\ V_0 = ?$

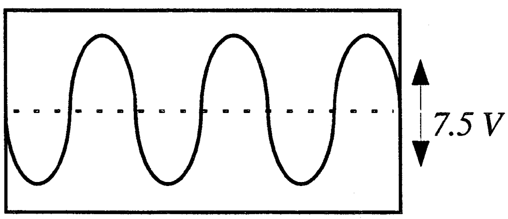
$$V_0 = -\frac{R_f}{R_1} V_1 = -\frac{100}{20} \times 1.5$$

$$V_0 = -7.5\ V$$

Peak value of output trace = 7.5 V.



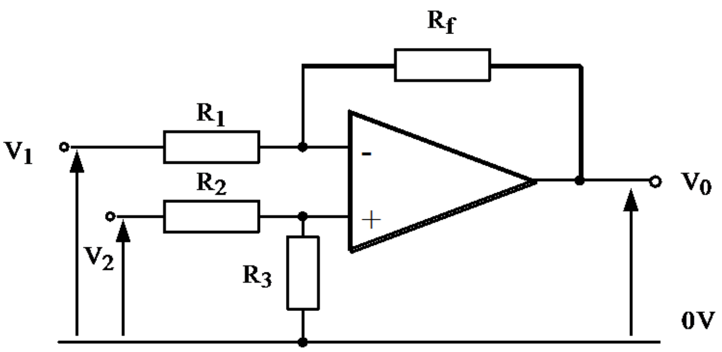
Input voltage V_1



Output voltage V_0 (inverted)

The Differential Amplifier

If the amplifier is set up in the configuration shown below, it is said to be in the **differential mode.**



There are two input potentials, V_1 and V_2, one applied to each of the input terminals of the op-amp.

There is a feedback resistor, R_f, connected between the output and the inverting input. This allows control over the gain of the amplifier as it did for the inverting mode.

When the op-amp is used in this mode, it amplifies the difference between the inputs V_1 and V_2, with a gain set by the ratio

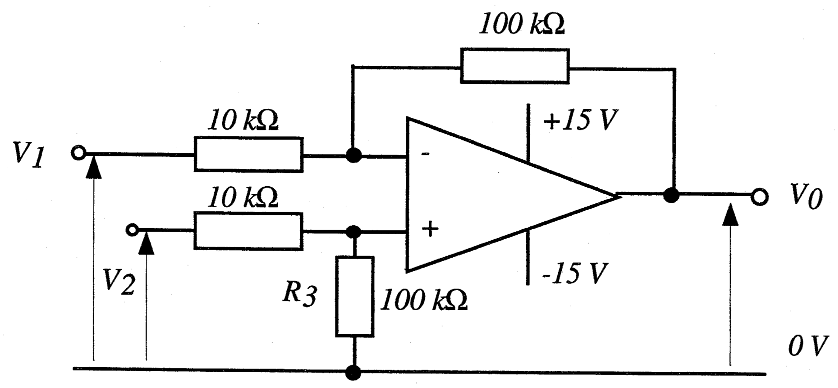
The formula for this amplifier is $\frac{R_f}{R_1}$.

$$V_0 = \frac{R_f}{R_1}(V_2 - V_1) \quad \text{providing} \quad \frac{R_f}{R_1} = \frac{R_3}{R_2}$$

V_0: Output potential; $\frac{R_f}{R_1}$: Gain; $(V_2 - V_1)$: Potential difference between inputs

For all questions that you will be asked on the differential amplifier: $\frac{R_f}{R_1} = \frac{R_3}{R_2}$

Example
A differential amplifier is set up as shown below.



For the following values shown, calculate the output voltage, V_0.

(a) $V_1 = +5.0$ V, $V_2 = +4.8$ V

(b) $V_1 = -2.0$ V, $V_2 = +4.5$ V

(a) $V_1 = +5.0$ V, $V_2 = +4.8$ V, $R_1 = R_2 = 10$ kΩ, $R_f = R_3 = 100$ kΩ, $V_0 = ?$

$$V_0 = \frac{R_f}{R_1}(V_2 - V_1) \qquad V_0 = \frac{100}{10}(4.8 - 5.0)$$

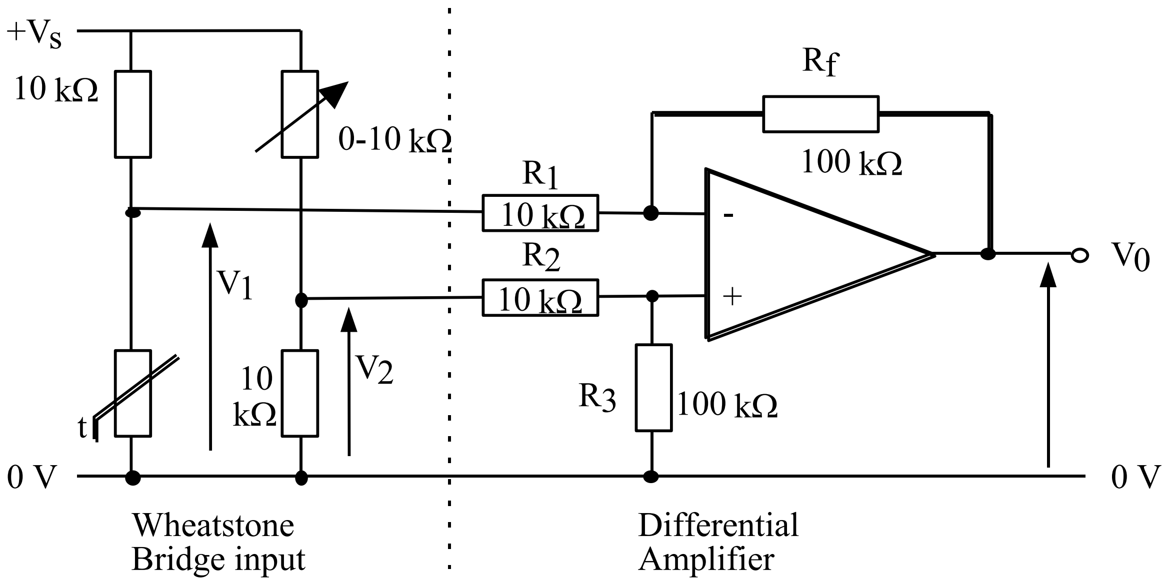
$$V_0 = -2.0 \text{ V}$$

(b) $V_1 = -2.0$ V, $V_2 = +4.5$ V, $R_1 = R_2 = 10$ kΩ, $R_f = R_3 = 100$ kΩ, $V_0 = ?$

$$V_0 = \frac{R_f}{R_1}(V_2 - V_1) \qquad V_0 = \frac{100}{10}(4.5 - (-2.0)) \qquad V_0 = +65 \text{ V in theory.}$$

But supply voltage = + 15 V hence output voltage, V_0, will saturate at + 15 V

The differential amplifier as part of a monitoring system



The setting of the variable resistor can be adjusted so as to achieve an output voltage of zero for a particular temperature setting. The bridge circuit would then be **balanced**, that is the potential difference $\mathbf{V_2 - V_1 = 0\ V}$.

The potential, V_2, will remain constant as long as the resistance of the variable resistor is not changed. Any change in temperature will change the potential, V_1, and will therefore produce a potential difference between V_2 and V_1.

The amplifier will then amplify the difference between V_2 and V_1, giving an output potential, V_0. The output voltage will increase as the change in thermistor resistance, ΔR_t, increases. The amplifier is, effectively, amplifying the out-of-balance potential difference from the Wheatstone Bridge.

Practical Application

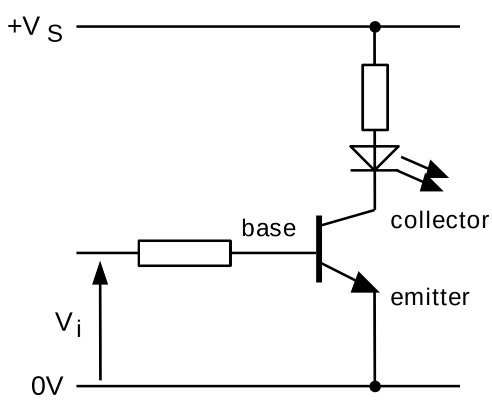
The output voltage could be calibrated by placing the thermistor in melting ice (0 ^oC), then in boiling water (100 ^oC), noting the output potential, V_0, for each case. The range could then be divided into 100 equal divisions to give an electronic thermometer over the range 0 - 100 ^oC.

Control Circuits

A transistor, such as those shown below, can act as an electrical switch.
If the input voltage to these transistors, V_i, is **positive**, then it switches on allowing a current to flow between the collector and emitter or source and drain, otherwise it is off.

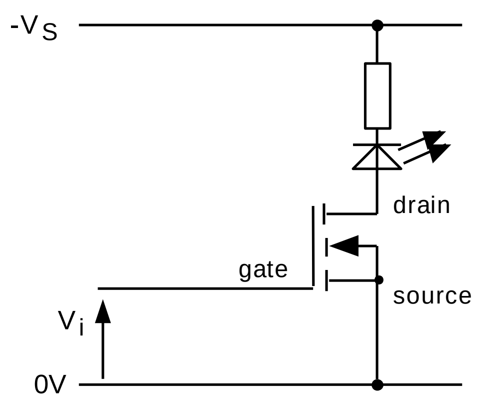
There are two types of transistor switches:

bipolar n-p-n transistor



(i) A positive (+ve) potential (>+0.7 V) is needed for the input, V_i, to switch transistor **on**.
(ii) The collector arm is connected to a positive (+ve) supply rail.
(iii) If the input potential is negative (<0.7 V), the transistor is off.

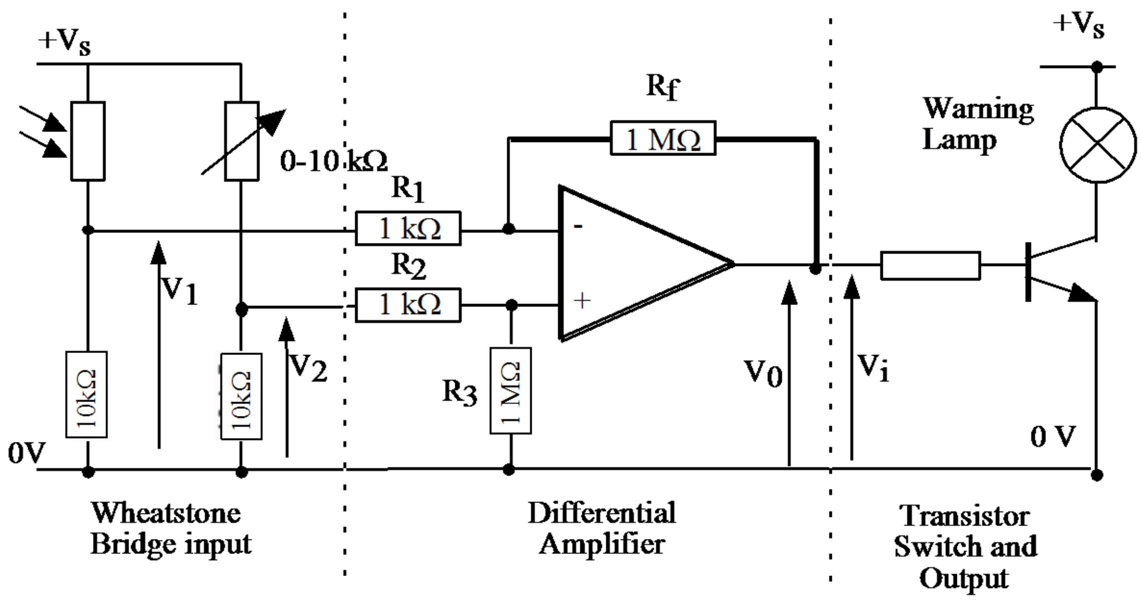
n-channel enhancement MOSFET



(i) A positive (+ve) potential >1.8 V is needed for the input, V_i, to switch transistor **on**.
(ii) The drain is connected to a positive (+ve) supply rail.
(iii) if the input potential is negative or <1.8 V the transistor is off.

These transistors can be used with a Wheatstone Bridge/differential amplifier circuit to switch a device on or off.

Low light-level indicator



The variable resistor in the Wheatstone Bridge is adjusted so that, at a particular light level, the output of the op-amp is zero or less so that the transistor is off. In practice, the variable resistor would be adjusted until the warning lamp is just off. In this situation, $V_1 \approx V_2$.
If the light level falls, the resistance of the LDR will **increase** causing the voltage V_1 to **fall**.
If V_1 falls, then $V_1 < V_2$, therefore $(V_2 - V_1)$ will be **positive** (+ve).
This will cause the **output** voltage, V_0, to be **positive** also, **switching on the transistor** and the **Warning Lamp lights**.

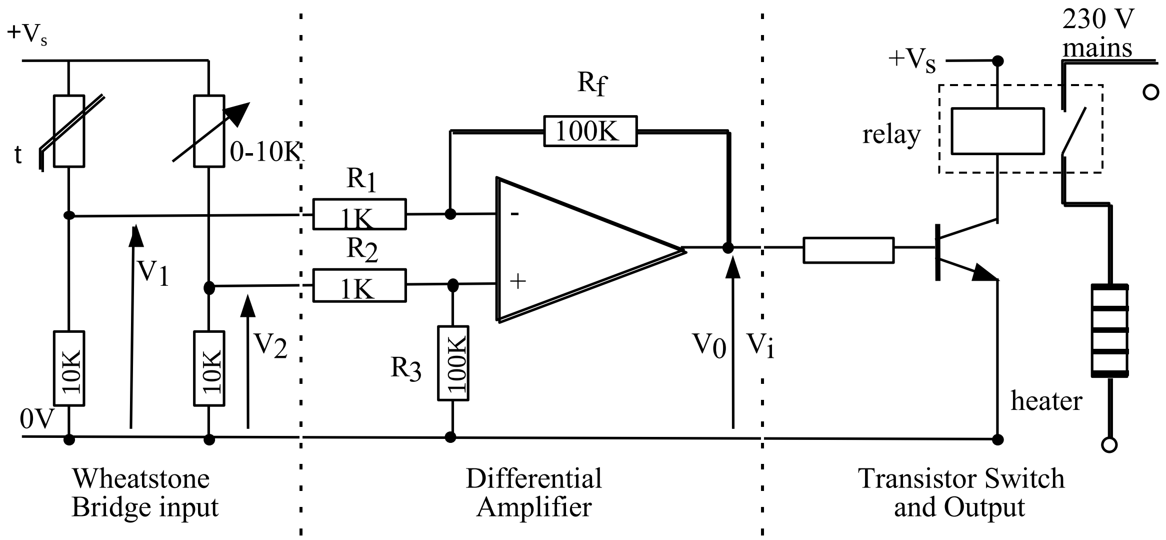
If the light level to the LDR increases, then the opposite will be true.
The resistance of the LDR will decrease causing the voltage V_1 to increase.
If V_1 increases, then $V_1 > V_2$, $(V_2 - V_1)$ will be negative causing the output to be negative and the transistor will not switch on.

The gain of the circuit (about 1000) is deliberately chosen to be large so that a small variation from the balance point of the Wheatstone Bridge produces a large enough output voltage to switch the transistor on.

Modifications

This circuit does have its limitations however. If the output device to be used has a high power rating requiring a current larger than the transistor can safely supply, then a **relay switch** must be used with the transistor to switch on an external circuit.

An example of this type of switch is shown below. In this case, a relay can be energised by the current through the transistor (the collector current) and its contacts can then be used to switch on a high-power device such as a heater.

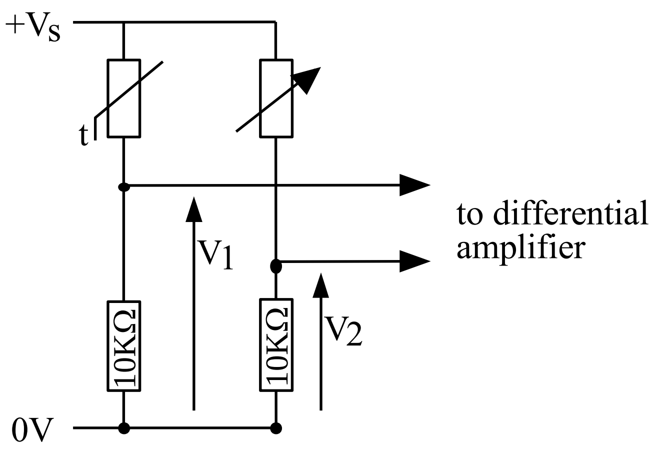


Notice that as the temperature falls the resistance of the thermistor increases, causing V_1 to fall. this means $V_1 < V_2$ giving a positive output voltage V_0. The transistor will be switched on, which will energise the relay switch. The heater will be turned on.

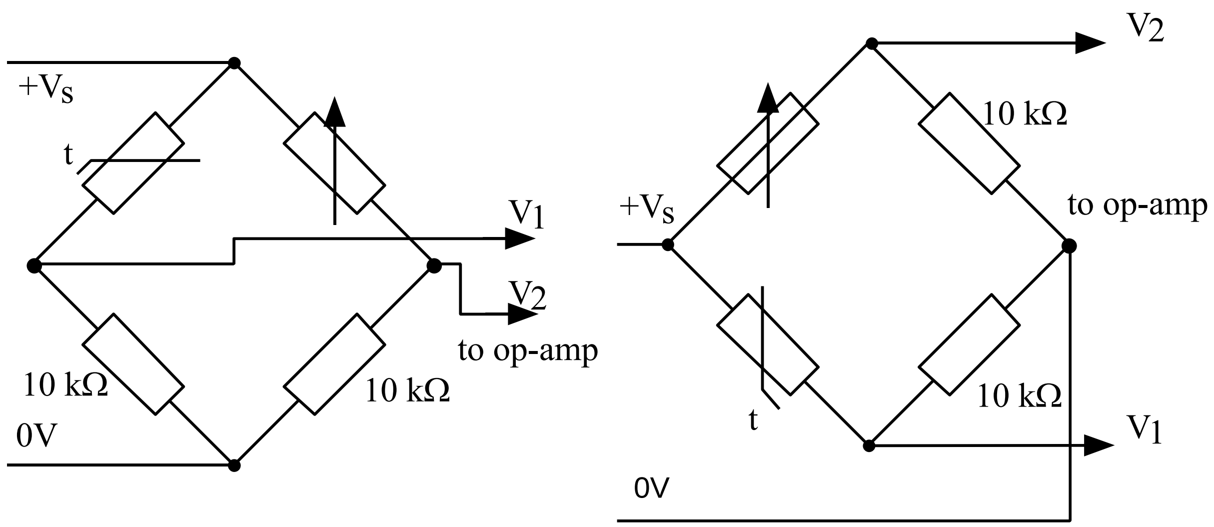
Control Circuit Diagrams

The Wheatstone Bridge arrangement can be shown as two straight potential dividers or as a diamond arrangement. They are **the same circuit**, just drawn a different way.

Straight potential dividers



Diamond arrangement bridge circuit



All the above circuits are identical examples of the sensor bridge circuit that can be used with a differential amplifier. It is important to recognise each circuit for what it is and how it behaves.